LECTURE 1 0F 3

TOPIC : 7.0 PERMUTATIONS AND COMBINATIONS

SUBTOPIC : 7.1 Permutations

LEARNING

OUTCOMES: At the end of the lesson students should be able to:

a) Use the techniques of counting.
b) Find permutations of a set of objects
c) Find the number of permutations of *n* different objects.
d) Find the number of permutations of *r* objects from *n* different objects.
e) Find the number of permutations of *n* objects comprising of r_1 identical objects, r_2 identical objects,…, r_k identical objects.

Definition of Permutation

A permutation is an arrangement of a group of objects in a particular order. The order of the objects is taken into consideration.

For example there are 6 permutations of letters A, B and C.
ABC CBA BAC CAB ACB BCA

a) Technique of counting (Multiplication Principle)

If the first event can take place in *m* ways and the second event can take place in *n* ways, therefore both the consecutive events can take place in *m x n* ways.

This principle can be extended to more than two consecutive events.

$$m_1 \; x \; m_2 \; x \; m_3 \; x \; m_4 \; x \ldots x \; m_n$$

Example 1:

In a computer game, there are 2 tunnels leading from point A to point B, 3 tunnels from point B to point C and 4 tunnel from point C to point D. Calculate the number of routes to move

a. From point A to point C via point B.
b. From point A to point D via points B and C.

Solution:

Example 2:

A shop stocks T – shirts in three sizes: small, medium and large. They are available in four colours: black, red, yellow and green. Find the number of different labels of colours and sizes.

Solution:

b) Permutations of n different objects

Number of permutations of *n* different objects taken all at a time without repetition is

$^{n}P_{n} = n! = nx(n-1)x(n-2)x...x2x1$

Example 3

There are 3 pictures to be hung in a line on a wall. In how many ways can they be arranged?

Solution:

Notes:

- 0! = 1
- $^{n}P_{n} = n!$ means the products of all the integers from 1 to *n* inclusive and is called '*n* factorial'.

Example 4:

How many different word codes can be formed from all the letters of the word 'LOGARITHM'?

Solution:

Example 5:

How many ways can 5 people be seated on 5 chairs arranged in a line

a. if there is no condition imposed?
b. if the first chair is reserved to one particular person?

Solution:

Example 6:

How many different 4 digit numbers can be formed from the digits 5, 6, 7 and 8.

a. with no repetitions.

b. if the first digit must be 7 and no repetition is allowed.

Solution:

c) Permutations of *r* objects taken from *n* different objects

The number of permutations of n different items taken r at a time is ${}^{n}P_{r}$ where

$$ {}^{n}P_{r} = \frac{n!}{(n-r)!} \quad \text{where } r \le n $$

If there is no restriction (repetition is allowed), the number of ways is n^{r}

Example 7:

In how many ways can the letter H, A, S, M, and R be arranged without repetition when

a. All the 5 letters are taken at a time.

b. 4 of the letters are taken at a time.

c. 2 of the letters are taken at a time.

Solution:

Example 8:

Suppose you have 4 different flags. How many different signals could you make using

a. 2 flags

b. 2 or 3 flags

Solution:

Example 9:

Four sisters and two brothers are arranged in different ways in a straight line for several photographs to be taken. How many different arrangements are possible if

(a) there are no restrictions

(b) the two brothers must be separated

Solution:

Example 10:

Find the number of arrangements of 4 digits taken from the set { 1, 2, 3, 4}

In how many ways can these numbers be arranged so that

(a) The numbers begin with digit '1'

(b) The numbers do not begin with digit '1'

Solution:

Example 11:

Four-digit numbers are to be formed from the digits 0, 1, 2, 3, 4, 5, 6 without repetition.

How many numbers can be formed if each number

a) Is less than 5000?

b) Begins with digit 4 or 6

c) Is between 2000 and 6000

d) Is an odd number

Solution:

Example 12:

How many three-digit numbers can be made from the integers 2, 3, 4, 5, 6 if

(i) Each integer is used only once?

(ii) There is no restriction on the number of times each integer can be used?

Solution:

Example 13:

In how many ways can 4 girls and 5 boys sit in a row if the boys and girls must sit alternate to each other?

Solution:

Example 14:

Three married couples have bought 6 seats in the same row for a concert. In how many different ways can they be seated

a) With no restrictions

b) If each couple is to sit together

c) If all the men sit together to the right of all the women

Solution:

Example 15:

There are 10 students out of whom six are females. How many possible arrangements are there if

a) They are arranged in a row?

b) Males always sit on one side and female on the other side?

Solution:

Example 16:

How many four-digit **even** numbers can be formed from the digits 0, 1, 2, 3, 4, 5, 6, 7 to make up numbers between 2000 and 6000

a) without repetition

b) with repetition

Solution:

d) Permutations of *n* objects comprising of r_1, r_2 ,…, r_k identical objects.

The number of permutations of *n* objects comprising of r_1 identical objects, r_2 identical objects, ………., r_k identical objects is $\dfrac{n!}{r_1!\,r_2!\,.........r_k!}$

Identical objects are objects that cannot be distinguished like identical twins, letters P in APPLE and so on.

Example 17:

Find the number of permutations of the letters D E F E A T E D.

Solution:

Example 18:

How many different permutations can be made using the letters of the words?

(i) LOTTO (ii) MATHEMATICS

Solution:

Example 19:

There are 2 copies of each of 3 different books to be arranged on a shelf. In how many distinguishable ways can this be done?

Solution:

Example 20:

How many different arrangements are there for the letters of the word **ARRANGEMENTS** if

a) begins with "R" and ends with "E"
b) the two letters "E" are separated
c) the two letters "E" and the two letters "A" are together
d) the consonant letters GMTS are together
e) the two letters "N" occupied both ends

Solution:

Example 21:

In how many ways can the word SEPTEMBER be arranged if

a) there is no restriction

b) all letters E are together

c) all letters E are separated.

Solution:

LECTURE 2 OF 3

TOPIC : 9.0 PERMUTATIONS AND COMBINATIONS

SUBTOPIC : 9.2 Combinations

LEARNING OUTCOMES : At the end of the lesson students should be able to:

a. Find the combination of a set of objects.
b. Determine the number of ways to form combinations of *r* objects from *n* objects.

Combinations of a set of objects

Definition

A combination is a **selection** of n distinct objects with no consideration given to the order of the objects.

- There is a difference between a permutation and a combination. In a permutation the order of the objects is important, whereas, in a combination the order is not important.
- For example, from three pictures A, B and C, the permutations of two objects are AB, AC, BC, BA, CA, and CB. Here $AB \neq BA$, $\mathrm{BC} \neq \mathrm{CB}$, $\mathrm{CA} \neq \mathrm{AC}$. Therefore, the total number of permutation is 6. However, in combination the **order is not important**, therefore AB = BA, BC = CB and AC = CA. Therefore, the total number of combination is only 3.

Example 1:

Determine whether each of the following is a permutation or combination:

a. 5 pictures **placed in a row**.
b. 3 story books picked from a rack.
c. A team of 9 players chosen from a group of 20.
d. The **arrangements** of the letters in the word OCTOBER.
e. Types of food in a plate taken for lunch consist of rice, vegetables, chicken curry and prawn paste sambal.

Therefore, we can conclude that **permutations** are used when **order is important** and **combinations** are used when **order is not important**.

Combinations of *r* objects from *n* different objects

- The number of combinations of *n* different objects taken r at a time is

$$ {}^{n}C_{r} \quad where \ {}^{n}C_{r} = \frac{n!}{r!(n-r)!} $$

Example 2:

There are 6 bottles of liquid dyes, consisting of the colours red, green, blue, yellow, white and black. Calculate the number of selections of the mixture of two different colours from the available six colours.

Solution:

Example 3:

A bicycle shop owner has 12 mountain bicycles in a showroom. The owner wishes to select 5 of them to display at a bicycle show. How many different ways can a group of 5 be selected?

Solution:

Example 4:

A quiz team of four is chosen from a group of 15 students. In how many ways could the team be chosen?

Solution:

Example 5:

In a test, a candidate is required to answer eight out of ten questions. Find the number of ways a candidate

a. Can answer the questions.
b. can answer the questions if the first three questions must be answered,

Solution:

Example 6:

In a public speaking competition, 7 out of the 11 contestants are female. Find the number of ways of choosing

a) the first place, the second place and the third place.
b) four winners consisting of at least two females.
c) two female and one male winners for the first place, the second place and the third place.

Solution:

LECTURE 3 OF 3

TOPIC : 9.0 PERMUTATIONS AND COMBINATIONS

SUBTOPIC : 9.2 Combinations

LEARNING OUTCOMES : At the end of the lesson students should be able to:

a. Determine the number of ways to form combinations of *r* objects from *n* objects.

Example 7:

If there are eight girls and seven boys in a class, in how many ways could a group be chosen so that there are two boys and two girls in the group?

Solution:

Example 8:

A 3 member committee is to be formed from 4 couples of husband and wife. Find the possible number of committees that can be formed if

a. All the members are men
b. The husband and the wife cannot be in the committee at the same time.

Solution:

Example 9:

A school committee consists of six girls and four boys. A social sub-committee consisting of four students is to be formed. In how many ways could the group be chosen if there are to be more girls than boys in the group?

Solution:

Example 10:

Form a group of chess players which consists of 4 male students and 3 female students, a school team which consists of 3 students is to be formed. Calculate the number of ways the team can be formed if

a. It must consist of exactly 2 males.
b. The number of females must be more than the number of males.

Solution:

Example 11:

In a football training squad of 24 people, 3 are goalkeepers, 7 are defenders, 6 are midfielders and 8 are forwards. A final squad of 16 selected for a match must consist of 2 goalkeepers, 4 defenders, 5 midfielders and 5 forwards. Find the number of possible selections if one particular goalkeeper, 2 particular defenders, 3 particular midfielders and 3 particular forwards are automatically selected.

Solution:

Example 12:

ABCDEFGH is a regular octagon.

a. How many triangles can be formed with the vertices of the octagon as vertices?
b. How many diagonals can be drawn by joining the vertices?

Solution:

Example 13:

A group of 12 environmentalists comprising of 7 engineers and 5 biologists are to be selected to form a committee. Find the number of ways of

a) forming a committee of 5 members consisting of at least 3 engineers

b) arranging all the 12 environmentalists in a row such that they are always in the group of the same expertise

c) selecting 3 members for the post of chairman, secretary and treasurer

d) forming a committee of 5 members with an engineer as chairman and a biologist as secretary.

Solution: