1

Let $u = 1 + e^x$

$\frac{du}{dx} = e^x$, $du = e^x\,dx$

When

$x = 0$; $u = 2$

$x = 1$; $u = 1 + e$

$$\int_2^{1+e} \frac{1}{u}\,du = [\ln u]_2^{1+e}$$

$$= \ln(1+e) - \ln 2$$

$$= \ln\left(\frac{1+e}{2}\right)$$

2

$$\int_1^2 x^2 \ln 3x\,dx$$

$u = \ln 3x$, $u' = \frac{1}{x}$

$v' = x^2\,dx$, $v = \frac{x^3}{3}$

$$\Rightarrow \int_1^2 x^2 \ln 3x\,dx = \frac{x^3}{3}(\ln 3x) - \int \frac{1}{x}\left(\frac{x^3}{3}\right)dx$$

$$= \frac{x^3 \ln 3x}{3} - \int \frac{x^2}{3}\,dx$$

$$= \left[\frac{x^3 \ln 3x}{3} - \frac{x^3}{9}\right]_1^2$$

$$= \left(\frac{8 \ln 6}{3} - \frac{8}{9}\right) - \left(\frac{\ln 3}{3} - \frac{1}{9}\right)$$

$$= 3.634$$

3

$$\frac{dy}{dx} = \frac{y}{2(x-1)}$$

$$\int \frac{1}{y}\,dy = \int \frac{1}{2(x-1)}\,dx$$

$$\ln y = \frac{1}{2}\ln(x-1) + c$$

$$y = e^{\ln(x-1)^{\frac{1}{2}} + c}$$

$$y = Ae^{\ln(x-1)^{\frac{1}{2}}}$$

$$y = A(x-1)^{\frac{1}{2}}$$

$x = 5, y = 2,$

$$2 = A(5-1)^{\frac{1}{2}}$$

$$2 = 2A$$

$$A = 1$$

$$\therefore y = (x-1)^{\frac{1}{2}} = \sqrt{x-1}$$

4

(a) Let $f(x) = e^x + x - 4$

$$f(1) = e^{(1)} + (1) - 4$$

$$= -0.2817 < 0$$

$$f(2) = e^{(2)} + (2) - 4$$

$$= 5.389 > 0$$

or $f(x) = -e^x - x + 4$

$f(1) = 0.2817$

$f(2) = -5.389$

Sign changed, therefore root lies between 1 and 2.

(b) Let $f(x) = e^x + x - 4$

$f'(x) = e^x + 1$ 9

$$x_0 = 1.2$$

$$x_1 = 1.2 - \frac{e^{1.2} + 1.2 - 4}{e^{1.2} + 1} = 1.07961$$

$$x_2 = 1.07961 - \frac{f(1.07961)}{f'(1.07961)} = 1.07374$$

$$x_3 = 1.07373$$

$$x_4 = 1.07373$$

$\therefore$ The root is 1.0737

5

(a)

$x = y^2$................(1)

$y = -x + 2$...........(2)

$(1) = (2) : y^2 = 2 - y$

squaring both sides

$y^2 + y - 2 = 0$

$(y-1)(y+2) = 0$

$y = 1,\ x = 1$

$y = -2,\ x = 4$

Intersection points are $(1,1)$ and $(4,-2)$

$$Area = \left| \int_{-2}^{1} (2-y)\,dy - \int_{-2}^{1} (y^2)\,dy \right|$$

$$= \left| \left[2y - \frac{y^2}{2} \right]_{-2}^{1} - \left[\frac{y^3}{3} \right]_{-2}^{1} \right|$$

$$= \left| \left[\left(2 - \frac{1}{2}\right) - \left(-4 - \frac{4}{2}\right) \right] - \left[\frac{1}{3} - \frac{-8}{3} \right] \right|$$

$$= \frac{9}{2}\ unit^2$$

(b)

$$\text{Volume} = \pi \int_{-3}^{-1} \left(x^2 + 3x\right)^2 dx$$

$$= \pi \int_{-3}^{-1} \left(x^4 + 6x^3 + 9x^2\right) dx$$

$$= \pi \left[\frac{x^5}{5} + \frac{6x^4}{4} + \frac{9x^3}{3} \right]_{-3}^{-1} = \pi \left[\frac{x^5}{5} + \frac{3x^4}{2} + 3x^3 \right]$$

$$= \pi \left[\left(-\frac{1}{5} + \frac{3}{2} - 3 \right) - \left(-\frac{243}{5} + \frac{243}{2} - 81 \right) \right]$$

$$= \frac{32}{5}\pi \text{ unit}^3$$ @ 20.1 unit³

6

(a) $\sqrt{(4-10)^2 + (-2-6)^2} = \sqrt{(a-4)^2 + (8+2)^2}$

$36 + 64 = (a-4)^2 + 100$

$(a-4)^2 = 0 \rightarrow a = 4$

Radius, r = $\sqrt{(4-4)^2 + (8+2)^2} = \sqrt{100} = 10$

The equation of circle : $(x-4)^2 + (y+2)^2 = 10^2$

$x^2 + y^2 - 8x + 4y - 80 = 0$

(b) Standard equation of parabola: $(y-2)^2 = 4p(x-3)$

At point $(4,4)$: $(4-2)^2 = 4p(4-3)$

$4 = 4p \rightarrow p = 1$

$(y-2)^2 = 4(1)(x-3)$

$(y-2)^2 = 4(x-3)$