

1. Using the by part integration, and assuming  $u = xe^x$  and  $dv = \frac{1}{(x+1)^2}$ , find

$$\int_0^1 \frac{xe^x}{(x+1)^2} dx.$$

[5 marks]

2. Find

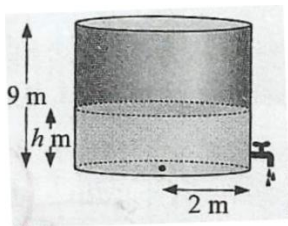
(a)  $\int xe^{4x} dx$

[3 marks]

(b)  $\int x\sqrt{x-2} dx$

[3 marks]

- 3.



Water flows out of a tap at the bottom of a large cylindrical tank at a rate proportional to the square root of the depth of the water remaining in the tank. Initially the tank is full to a depth of 9m. After 15 minutes the depth of water is 4m. How long would it take for the tank to empty?

[7 marks]

4. Show that the equation  $\tan x = 2x$  has a root in the interval  $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ . Use Newton-

Raphson to find the root of the equation between  $\frac{\pi}{4}$  and  $\frac{\pi}{2}$ , correct to three decimal places.

[10 marks]

5. Prove that  $16x^2 + 9y^2 - 32x + 36y - 92 = 0$  is an ellipse. Hence find the coordinates for its vertices and centre. Sketch the graph of the ellipse.

[10 marks]

6. (a) Find the area of the region bounded by the curve  $y = \frac{x^2}{2} + 1$  and the graph

$$y = \frac{3}{2}|x| + 1.$$

**[6 marks]**

- (b) Find the volume generated when the region bounded by the curve  $y = \frac{x^2}{2} + 1$  and the graph  $y = \frac{3}{2}|x| + 1$  for  $x > 0$  is rotated completely about the  $y$ -axis.

**[6 marks]**

**END OF QUESTION**

**Final Answer**

1.  $\frac{e}{2} - 1$

2. (a)  $\frac{xe^{4x}}{4} - \frac{e^{4x}}{16} + c$

(b)  $\frac{2}{5}(x-2)^{\frac{5}{2}} + \frac{4}{3}(x-2)^{\frac{3}{2}} + c$

3. 45 minutes

4. 1.166

5. centre  $(1, -2)$  ; vertices  $(1, -2 \pm 4)$ 

6. (a)  $4\frac{1}{2}$  unit<sup>2</sup>

(b)  $6\frac{3}{4}\pi$  unit<sup>3</sup>