1.

$a)\quad x^2 + y^2 - 2x - 8y + 16 = 0$

completing the square:

$(x-1)^2 + (y-4)^2 = 1$

center : $(1,4)$ *radius* $= 1$

$b)\quad$ *Substitute* $A(4,8)$ *into the standard eq*;

$(4-1)^2 + (8-4)^2 = 25 \quad (>1)$

So, the point lies outside of the circle.

length of tangent

$= \sqrt{4^2 + 8^2 - 2(4) - 8(8) + 16}$

$= \sqrt{24}$

$= 2\sqrt{6}$ *unit*

2. $y = e^{3x} \ln x$

$$\frac{dy}{dx} = e^{3x}\left(\frac{1}{x}\right) + \ln x\left(3e^{3x}\right)$$

$$= e^{3x}\left[\frac{1}{x} + 3\ln x\right]$$

$$\int \frac{e^{3x}}{9}\left[\frac{1}{x} + 3\ln x\right]dx$$

$$= \frac{1}{9}\int e^{3x}\left[\frac{1}{x} + 3\ln x\right]dx$$

$$= \frac{1}{9}\int \frac{dy}{dx}\ dx$$

$$= \frac{1}{9}\ y + c$$

$$= \frac{1}{9}e^{3x} \ln x + c$$

3. $f(x)=e^{2x}-16+16\cos^2 x \;\rightarrow\; f'(x)=2e^{2x}-32\sin x\cos x$

$f(1)=e^{2}-16+16\cos^2(1)=-3.94<0$

$f(2)=e^{4}-16+16\cos^2(4)=45.43>0$

Since f(1) and f(2) have opposite signs, $e^{2x}=16-16\cos^2 x$ has a root between $x=1$ and $x=2$.

Using $x_1=1.4$

$$x_2=1.4-\frac{f(1.4)}{f'(1.4)}=1.3671$$

$$x_3=1.3671-\frac{f(1.3671)}{f'(1.3671)}=1.3650$$

$$x_4=1.3650-\frac{f(1.3650)}{f'(1.3650)}=1.3650$$

$$x_5=1.3650-\frac{f(1.3650)}{f'(1.3650)}=1.3650$$

Therefore, the solution of $e^{2x}=16-16\cos^2 x$ is 1.37 (3 s.f.)

b) $\frac{dy}{dx}=\ln(5-x)-\frac{x}{5-x}$

$when\ \frac{dy}{dx}=0 \quad \Rightarrow \ln(5-x)=\frac{x}{x-5}$

$5-x=e^{\frac{x}{5-x}}$

$x=5-e^{\frac{x}{5-x}} \quad (shown)$

c) $x_1=5-e^{\frac{2.4}{5-2.4}}=\ 2.483$

$$x_2 = 5 - e^{\frac{2.483}{5-2.483}} = \quad 2.318 = \ 2.32 \ \ (3 \text{ s.f})$$

4. a) $$(x+1)\frac{dy}{dx} - y = x^2 - 1$$

$$\frac{dy}{dx} - \frac{y}{x+1} = x - 1$$

$$\therefore \ \ p(x) = -\frac{1}{x+1}$$

$$\begin{aligned} V(x) &= e^{\int -\frac{1}{x+1}dx} \\ &= e^{-\ln|x+1|} \\ &= \frac{1}{x+1} \end{aligned}$$

$$\frac{dy}{dx} - \frac{y}{x+1} = x - 1$$

$$\frac{1}{x+1}\frac{dy}{dx} - \frac{y}{(x+1)^2} = \frac{x-1}{x+1}$$

$$\frac{d}{dx}\left(\frac{y}{x+1}\right) = \frac{x-1}{x+1}$$

$$\begin{aligned} \frac{y}{x+1} &= \int \frac{x-1}{x+1}dx \\ &= \int 1 - \frac{2}{x+1}dx \\ &= x - 2\ln|x+1| + c \\ y &= (x+1)[x - 2\ln|x+1| + c] \end{aligned}$$

b) $$(2x^2 + x)\frac{dy}{dx} = \frac{4x+1}{\tan y} \quad ; y(1) = 0$$

$$\int \tan y dy = \int \frac{4x+1}{2x^2 + x}dx$$

$$-\ln(\cos y) = \ln(2x^2 + x) + C$$

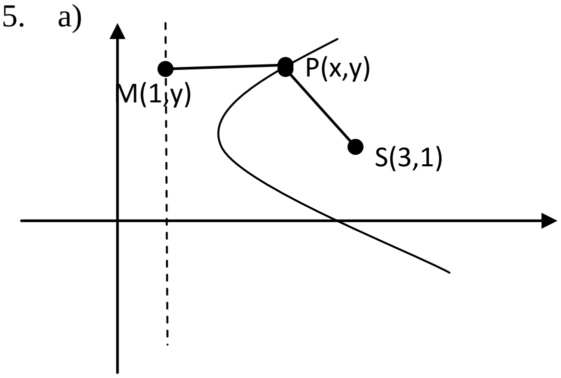
$$\ln[(2x^2 + x)(\cos y)] = C$$

$$y(1) = 0 \Rightarrow C = \ln[(3)(\cos 0)] = \ln 3$$

$$(2x^2 + x)(\cos y)] = 3$$

$$\therefore\ y = \cos^{-1}\left(\frac{3}{2x^2 + x}\right)$$

5. a)



$$PM = PS$$
$$\sqrt{(x-1)^2 + (y-y)^2} = \sqrt{(x-3)^2 + (y-1)^2}$$
$$(x-1)^2 = (x-3)^2 + (y-1)^2$$
$$x^2 - 2x + 1 = x^2 - 6x + 9 + y^2 - 2y + 1$$
$$y^2 - 2y - 4x + 9 = 0$$

$b)$ $\dfrac{(x+1)^2}{a^2} + \dfrac{(y-2)^2}{b^2} = 1$

At (4,2): $\dfrac{(4+1)^2}{a^2} + \dfrac{(2-2)^2}{b^2} = 1 \quad \Rightarrow a^2 = 25$

At $(-5,-\frac{2}{5})$: $\frac{(-5+1)^2}{25}+\frac{(-\frac{2}{5}-2)^2}{b^2}=1 \quad \Rightarrow b^2=16$

$$\frac{(x+1)^2}{25}+\frac{(y-2)^2}{16}=1$$

$$16(x+1)^2+25(y-2)^2=400$$
$$16x^2+25y^2+32x-100y-284=0$$

6. $\frac{3x^2-7x+6}{(x-3)^2(x+1)}=\frac{A}{x+1}+\frac{B}{x-3}+\frac{C}{(x-3)^2}$

$$=\frac{A(x-3)^2+B(x+1)(x+3)+C(x+1)}{(x-3)^2(x+1)}$$

Comparing the numerator:

$$3x^2-7x+6=A(x-3)^2+B(x+1)(x-3)+C(x+1)$$

When $x=3$, 27-21+6=4C

C = 3

When $x=-1$, 3+7+6= 16A

A = 1

Comparing the coefficient of x^2:

3 = A + B

3 = 1 + B

B = 2

$$\therefore \frac{3x^2 - 7x + 6}{(x-3)^2(x+1)} = \frac{1}{(x+1)} + \frac{2}{(x-3)} + \frac{3}{(x-3)^2}$$

$$\int_1^2 \frac{3x^2 - 7x + 6}{(x-3)^2(x+1)}\ dx$$

$$= \int_1^2 \frac{1}{(x+1)}dx + \int_1^2 \frac{2}{(x-3)}dx + \int_1^2 \frac{3}{(x-3)^2}dx$$

$$= \int_1^2 \frac{1}{(x+1)}dx + \int_1^2 \frac{2}{(x-3)}dx + \int_1^2 3(x-3)^{2}\ dx$$

$$= \left[\ln|x+1|\right]_1^2 + \left[2\ln|x-3|\right]_1^2 - \left[\frac{3}{x-3}\right]_1^2$$

$$= \left[\ln(x+1)(x-3)^2\right] - \left[\frac{3}{x-3}\right]_1^2$$

$$= \ln 3 - \ln 8 - \left[-3 - \left(\frac{3}{-2}\right)\right]$$

$$= \frac{3}{2} + \ln\left(\frac{3}{8}\right)$$