

1. The equation of the circle is $x^2 + y^2 - 2x - 8y + 16 = 0$.
- Find the center and radius of the circle.
 - Show that the point A (4,8) lies outside of the circle. Hence find the length of the tangent to the circle from the point A.
- [5 marks]**
2. Given $y = e^{3x} \ln x$, find $\frac{dy}{dx}$. Hence find $\int \frac{e^{3x}}{9} \left[\frac{1}{x} + 3 \ln x \right] dx$.
- [6 marks]**
3. Show that the equation $e^{2x} = 16 - 16 \cos^2 x$ has a root between $x=1$ and $x=2$. Use the Newton-Raphson method to obtain the root of the equation, correct to three significant figures, by taking $x=1.4$ as the first approximation.
- [7 marks]**
4. Find the solutions of the following differential equations.
- $(x+1) \frac{dy}{dx} - y = x^2 - 1$ **[5 marks]**
 - $(2x^2 + x) \frac{dy}{dx} = \frac{4x+1}{\tan y}$; $y(1) = 0$ **[5 marks]**
5. a) Find the equation of the parabola whose set of points (x, y) are such that the distance of each point from (3,1) is equal to its distance from the line $x = 1$.
- [4 marks]**
- b) The center of an ellipse is (-1,2), the minor axis is parallel to the y-axis and passes through the point (4,2) and $(-5, -\frac{2}{5})$. Find the general equation of the ellipse.
- [6 marks]**
6. Express $\frac{3x^2 - 7x + 6}{(x-3)^2(x+1)}$ in the form of partial fraction. Hence evaluate $\int_1^2 \frac{3x^2 - 7x + 6}{(x-3)^2(x+1)} dx$. Give the answer in the form of $a + \ln b$.
- [12 marks]**

END OF QUESTION

Final Answer

1. a) center = (1,4) ; radius = 1 b) $2\sqrt{6}$

2. $\frac{dy}{dx} = e^{3x} \left[\frac{1}{x} + 3 \ln x \right]$; $\frac{1}{9} e^{3x} \ln x + c$

3. 1.37

4. a) $y = (x+1)[x - 2 \ln|x+1| + c]$

b) $y = \cos^{-1} \left(\frac{3}{2x^2 + x} \right)$

5. a) $y^2 - 2y - 4x + 9 = 0$

b) $16x^2 + 25y^2 + 32x - 100y - 284 = 0$

6. a) $\frac{1}{x+1} + \frac{2}{x-3} + \frac{3}{(x-3)^2}$; $\frac{3}{2} + \ln \left(\frac{3}{8} \right)$