1. a)

$$\int_1^2 2x^2\sqrt{x^3+2}\ dx = \frac{2}{3}\int_3^{10}\sqrt{u}\ du$$

$$= \frac{4}{9}\left[\left(\sqrt{u}\right)^3\right]$$

$$= \frac{4}{9}\left[\left(\sqrt{10}\right)^3 - \left(\sqrt{3}\right)^3\right]$$

$$= 11.745$$

$u = x^3 + 3$

$\dfrac{du}{3} = x^2 dx$

b)

$$\int_0^1 xe^{-2x}dx \ = uv - \int v du$$

$$= \left[x\left(\frac{e^{-2x}}{-2}\right)\right]_0^1 - \int_0^1 \frac{e^{-2x}}{-2}dx$$

$$= -\frac{1}{2}\left(e^{-2} - 0\right) - \frac{1}{4}\left(e^{-2} - e^0\right)$$

$$= \frac{1}{4}(1 - 3e^{-2})$$

$u = x$ $\quad dv = e^{-2x}dx$

$du = dx$ $\quad v = \dfrac{e^{-2x}}{-2}$

2. N_1 : $\quad y + 1 = -\dfrac{3}{2}(x - 6)$

$$y = -\frac{3}{2}x + 8$$

N_2 : $\quad y - 0 = \dfrac{2}{3}(x - 1)$

$$y = \frac{3}{2}x - \frac{2}{3}$$

Centre = intersection N_1,N_2

$$-\frac{3}{2}x + 8 = \frac{2}{3}x - \frac{2}{3}$$

$$-13x = -\frac{26}{3}(6)$$

$x = 4 \quad ; \quad y = 2 \qquad \therefore C(4,2)$

Then, $r = \sqrt{(4-1)^2 + (2-0)^2} \qquad r = \sqrt{13}$

Equation of the circle : $(x-4)^2 + (y-2)^2 = 13 \quad or \quad x^2 + y^2 - 8x - 4y + 7 = 0$

3. $25x^2 + 9y^2 - 150x - 18y + 9 = 0$

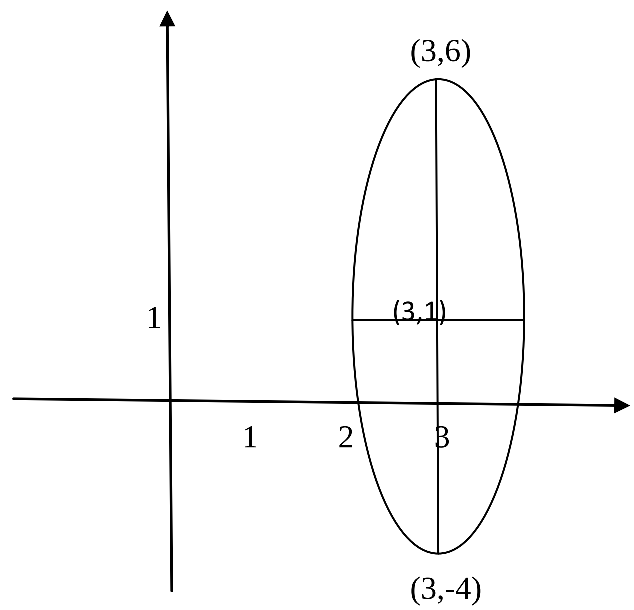
$25(x^2 - 6x) + 9(y^2 - 2y) + 9 = 0$

$25(x-3)^2 - 225 + 9(y-1)^2 - 9 + 9 = 0$

$$\frac{25(x-3)^2}{225} + \frac{9(y-1)^2}{225} = \frac{225}{225}$$

$$\frac{(x-3)^2}{9} + \frac{(y-1)^2}{25} = 1$$

Centre =(3 , 1) V = (3, 6) , (3, -4)



$$\cos ec\, x \frac{dy}{dx} + y \sec x = 2 \cos x$$

$\div \cos ecx$

4. $$\frac{dy}{dx} + \frac{\sin x}{\cos x} y = 2 \sin x \cos x$$

$$P(x) = \frac{\sin x}{\cos x} \quad , \quad Q(x) = 2 \sin x \cos x$$

i) $$V(x) = e^{\int \frac{\sin x}{\cos x} dx} = \frac{1}{\cos x}$$

ii) $$V(x)y = \int V(x)Q(x)dx$$

$$\frac{1}{\cos x} y = \int \frac{1}{\cos x} (2 \sin x \cos x) dx$$

$$y = \cos x(-2\cos x + k)$$

$$y = k\cos x - 2\cos^2 x$$

5. $$\int_0^2 y\,dx = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + \ldots.. + y_{n-1})\right]$$

$$= \frac{h}{2}\left[(0 + 1.6094) + 2\sum_{i=1}^{n-1} y_i)\right]$$

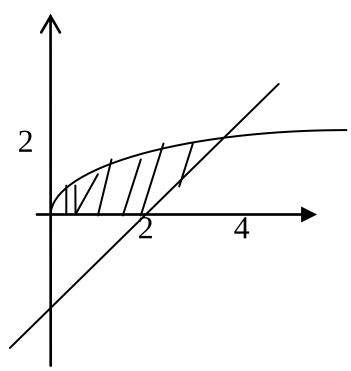
$$= \frac{h}{2}\left[1.6094 + 2(4.9446)\right]$$

$$1.4373 = \frac{h}{2}[11.4986]$$

$$h = 0.25 \quad where \quad h = \frac{b-a}{n}$$

$$\frac{2-0}{n} = 0.25 \quad \Rightarrow \quad n = 8$$

6. $$x - 2 = \sqrt{x}$$
$$x^2 - 5x + 4 = 0$$
$$x = 1 \, or \, 4$$



Area = $\int_0^4 \sqrt{x}dx - \frac{1}{2}(2)(2)$

$$= \left[\frac{2(x)^{\frac{3}{2}}}{3}\right]_0^4 - 2 = \frac{10}{3}\text{unit}^2$$

$$\text{Volume} = \pi\int_0^4 x\,dx - \pi\int_2^4 (x-2)^2\,dx$$

$$= \pi\left[\frac{x^2}{2}\right]_0^4 - \pi\left[\frac{x^3}{3} - 2x^2 + 4x\right]_2^4$$

$$= 8\pi - \pi\left(\left(\frac{64}{3} - 32 + 16\right) - \left(\frac{8}{3} - 8 + 8\right)\right)$$

$$= = \frac{16}{3}\pi \text{ cu.unit}^3$$