1. Evaluate

   a) $\displaystyle\int_{1}^{2} 2x^2 \sqrt{x^3 + 2}\,dx$ **[5 marks]**

   b) $\displaystyle\int_{0}^{1} xe^{-2x}\,dx$ leaving your answer in terms of e   **[4 marks]**

2. A circle touches two perpendicular lines $2x - 3y = 15$ and $3x + 2y = 3$ at points A $(6, -1)$ and B $(1, 0)$ respectively. Find the equation of the circle.

                                                                   [8 marks]

3. Show that the equation $25x^2 + 9y^2 - 150x - 18y + 9 = 0$ represents an ellipse and find the coordinates of the centre and vertices. Sketch the ellipse.   **[7 marks]**

4. Show that the differential equation $\cos ec\, x \dfrac{dy}{dx} + y\sec x = 2\cos x$ can be expressed in the

   form $\dfrac{dy}{dx} + P(x)y = Q(x),$ where P and Q are functions in x. Hence prove that the

   general solution of this differential equation is $y = c\cos x - 2\cos^2 x,$ where c is a

   constant.

                                                                    [7 marks]

5. Given that $y = \ln(1 + x^2),$ if an approximate of

   $\displaystyle\int_{0}^{2} y\,dx = 1.4373,$ $y_n = 1.6094$ and $\displaystyle\sum_{i=1}^{n-1} y_i = 4.9446.$ Find the value of n.

                                                                     [7 marks]

6. a) Find the area of the region bounded by $y = \sqrt{x}$, $y = x - 2$ and the x-axis.

   b) Hence, find the volume of the solid generated by revolving the region 2π radians about the x-axis. Give your answer in terms of π.

                                                                     [12 marks]

END OF QUESTION

Final Answer

1.   a) 11.745   b) $\frac{1}{4}(1 - 3e^{-2})$

2. $x^2 + y^2 - 8x - 4y + 7 = 0$

3. $\frac{(x-3)^2}{9} + \frac{(y-1)^2}{25} = 1$;   Centre : (3, 1)   Vertices : (3, -4), (3, 6)

5.   n = 8

6.   Area = $\frac{10}{3}$ unit^2, $V = \frac{16}{3}\pi$ cu. unit