1. axis parallel to $x$-axis

$(y-k)^2 = 4p(x-h)$

$V(h,\ k) = V(2,\ 1)$

$h = 2,\ k = 1$

$(y-1)^2 = 4p(x-2)$

At point (1, 0)

$(0-1)^2 = 4p(1-2)$

$p = -\frac{1}{4}$

$(y-1)^2 = 4(-\frac{1}{4})(x-2)$

$(y-1)^2 = -(x-2)$

2. $x^2 + y^2 + 2gx + 2fy + c = 0$

At (0, 8)

$0^2 + 8^2 + 2g(0) + 2f(8) + c = 0$

$16f + c = -64$ (1)

At (7, 9)

$7^2 + 9^2 + 2g(7) + 2f(9) + c = 0$

$14g + 18f + c = -130$ (2)

(2) – (1)

$7g + f = -33$ (3)

$4x - 3y + 24 = 0$

3y = 4x + 24

$y = \frac{4}{3}x + 8 \quad \Rightarrow m = \frac{4}{3}$

$m_N = -\frac{3}{4}$ at (0, 8)

Equation of normal

$y - 8 = -\frac{3}{4}(x - 0)$

$-f - 8 = -\frac{3}{4}(-g - 0)$

$f = -\frac{3}{4}g - 8$ (4)

Substituting (4) into (3)

$7g - \frac{3}{4}g - 8 = -33$

$g = -4$

(4) $f = -5$

(1) $c = 16$

General equation of circle

$x^2 + y^2 - 8x - 10y + 16 = 0$

3. $e^{-x}\dfrac{dy}{dx}=(1-y)^2$

$\dfrac{1}{e^x}\dfrac{dy}{dx}=(1-y)^2$

$\int\dfrac{1}{(1-y)^2}\,dy=\int e^x\,dx$

$\int(1-y)^{-2}\,dy=\int e^x\,dx$

$\dfrac{(1-y)^{-1}}{-1(-1)}=e^x+C$

$\dfrac{1}{1-y}=e^x+C$

$y=0\ ,\ x=0$

$\dfrac{1}{1-0}=e^0+C$

$C=0$

$\dfrac{1}{1-y}=e^x$

$1-y=\dfrac{1}{e^x}$

$y=1-e^{-x}$

4. $y=(\cos 2x-\sin 2x)^2$

$=\cos^2 2x+\sin^2 2x-2\cos 2x\sin 2x$

$=1-\sin 4x$

At $x$-axis, $y=0$

$1-\sin 4x=0$

$\sin 4x = 1$

$4x = \frac{\pi}{2}$

$x = \frac{\pi}{8}$

Area = $\int_0^{\frac{\pi}{8}} 1 - \sin 4x \; dx$

$= \left[ x + \frac{\cos 4x}{4} \right]_0^{\frac{\pi}{8}}$

$= \left[ \frac{\pi}{8} + \frac{0}{4} \right] - \left[ 0 + \frac{1}{4} \right]$

$= \frac{\pi}{8} - \frac{1}{4}$

$= 0.143 \; unit^2$

5. a) $\int (t-1)\ln t \; dt = (\ln t)\left( \frac{t^2}{2} - t \right) - \int \left( \frac{t^2}{2} - t \right)\left( \frac{1}{t} dt \right)$

$u = \ln t \qquad \int dv = \int (t-1) dt$

$\frac{du}{dt} = \frac{1}{t} \qquad v = \frac{t^2}{2} - t$

$= \left( \frac{t^2}{2} - t \right) \ln t - \int \frac{t}{2} - 1 dt$

$= \left( \frac{t^2}{2} - t \right) \ln t - \left( \frac{t^2}{4} - t \right) + C$

$= \left( \frac{t^2}{2} - t \right) \ln t - \frac{t^2}{4} + t + C$

b) $\int 4x \ln(2x+1) \; dx = \int 4\left( \frac{t-1}{2} \right) \ln t \left( \frac{dt}{2} \right)$

$t = 2x + 1$

$dt = 2dx$

$= \int (t-1)\ln t \; dt$

c) $\int_0^1 4x\ln(2x+1)\,dx = \int_1^3 (t-1)\ln t\,dt$

$t = 2x+1$
$x = 0,\ t = 1$
$x = 1,\ t = 3$

$$= \left[\left(\frac{t^2}{2} - t\right)\ln t - \frac{t^2}{4} + t\right]_1^3$$

$$= \left[\left(\frac{9}{2} - 3\right)\ln 3 - \frac{9}{4} + 3\right] - \left[\left(\frac{1}{2} - 1\right)\ln 1 - \frac{1}{4} + 1\right]$$

$$= \frac{3}{2}\ln 3$$

6. a) $x^3 + x - 5 = 0$

Let $f(x) = x^3 + x - 5$, $f'(x) = 3x^2 + 1$

when $x = 1$, $f(1) = (1)^3 + 1 - 5 = -3 < 0$

when $x = 2$, $f(2) = (2)^3 + 2 - 5 = 5 > 0$

the opposite sign of $f(1)$ and $f(2)$ shows that there is a root between $x = 1$ and $x = 2$.

Take $x_0 = 1$ as first approximation, by using Newton Raphson formula , hence

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{(1^3 + 1 - 5)}{3(1)^2 + 1} = 1.750$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.750 - \frac{(1.750^3 + 1.750 - 5)}{3(1.750)^2 + 1} = 1.543$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.543 - \frac{(1.543^3 + 1.543 - 5)}{3(1.543)^2 + 1} = 1.516$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.516 - \frac{(1.516^3 + 1.516 - 5)}{3(1.516)^2 + 1} = 1.516$$

Therefore the root is 1.52.

b) $\int_{-1}^{1} \sqrt{1-x^2}\,dx$, $n = 6$ $\quad h = \dfrac{b-a}{h} = \dfrac{1-(-1)}{6} = \dfrac{1}{3}$

| $x$ | $y = \sqrt{1-x^2}$ | |
|---|---|---|
| $x_0 = -1$ | $y_0 = 0$ | |
| $x_1 = -\frac{2}{3}$ | | $y_1 = 0.7454$ |
| $x_2 = -\frac{2}{3}$ | | $y_2 = 0.9428$ |
| $x_3 = 0$ | | $y_3 = 1$ |
| $x_4 = \frac{1}{3}$ | | $y_4 = 0.9428$ |
| $x_5 = \frac{2}{3}$ | | $y_5 = 0.7454$ |
| $x_6 = 1$ | $y_6 = 0$ | |
| Total | 0 | 4.3764 |

$$\int_{-1}^{1} \sqrt{1-x^2}\,dx \approx \frac{h}{2}\left[(y_0 + y_6) + 2(y_1 + y_2 + \ldots.y_6)\right]$$

$$= \frac{1}{6}\left[0 + 2(4.3764)\right]$$

$$= 1.459$$