1. Find the equation of the parabola with vertex at (2, 1) , the axis of symmetry is parallel to x-axis and passing through the point (1, 0). **[6 marks]**

2. Find the general equation of a circle that touches the straight line $4x-3y+24=0$ at the point (0, 8) and passes through the point (7, 9). **[7 marks]**

3. Given $e^{-x}\dfrac{dy}{dx}=(1-y)^2$ and $y=0$ when $x=0$, express y in terms of x. **[7 marks]**

4. Find the area of the region bounded by the curve $y=(\cos 2x-\sin 2x)^2$, the y-axis, and the x-axis from $x=0$ to the first point where the curve touches the positive x-axis. **[7 marks]**

5 a) Use integration by parts to find $\int (t-1)\ln t\ dt$. **[4 marks]**

b) Use the substitution $t=2x+1$ to show that $\int 4x\ln(2x+1)\ dx$ can be written as $\int (t-1)\ln t\ dt$ **[3 marks]**

c) Hence find the exact value of $\int_0^1 4x\ln(2x+1)\,dx$. **[4 marks]**

6 a) Show that the equation $x^3+x=5$ has a root between 1 and 2. By using the Newton-Raphson method and taking $x_0=1$ as a first approximation, determine an approximation to this root, giving your answer to two decimal places. **[6 marks]**

b) Use the trapezoidal rule to approximate $\int_{-1}^{1}\sqrt{1-x^2}\,dx$ with 6 subintervals, giving your answer correct to three decimal places. **[6 marks]**

END OF QUESTION

Final Answer

1. $(y-1)^2 = -(x-2)$

2. $x^2 + y^2 - 8x - 10y + 16 = 0$

3. $y = 1 - e^{-x}$

4. $0.143 \ unit^2$

5 a) $\left(\frac{t^2}{2} - t\right)\ln t - \frac{t^2}{4} + t + C$

c) $\frac{3}{2}\ln 3$

6 a) 1.52

b) 1.459