1. $\int_0^{\frac{\pi}{2}} \sin^2 x \cos^2 x\, dx = \int_0^{\frac{\pi}{2}} \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right) dx$

$$= \frac{1}{4}\int_0^{\frac{\pi}{2}} \left(1-\cos^2 2x\right) dx$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{2}} \left(1-\left[\frac{1+\cos 4x}{2}\right]\right) dx$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{2}} \left(\frac{1}{2}-\frac{1}{2}\cos 4x\right) dx$$

$$= \frac{1}{8}\int_0^{\frac{\pi}{2}} \left(1-\cos 4x\right) dx$$

$$= \frac{1}{8}\left[x-\frac{\sin 4x}{4}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{8}\left[\left(\frac{\pi}{2}-0\right)-0\right]$$

$$= \frac{\pi}{16}$$

2. $x^2 - 4x + 6y - 2 = 0$

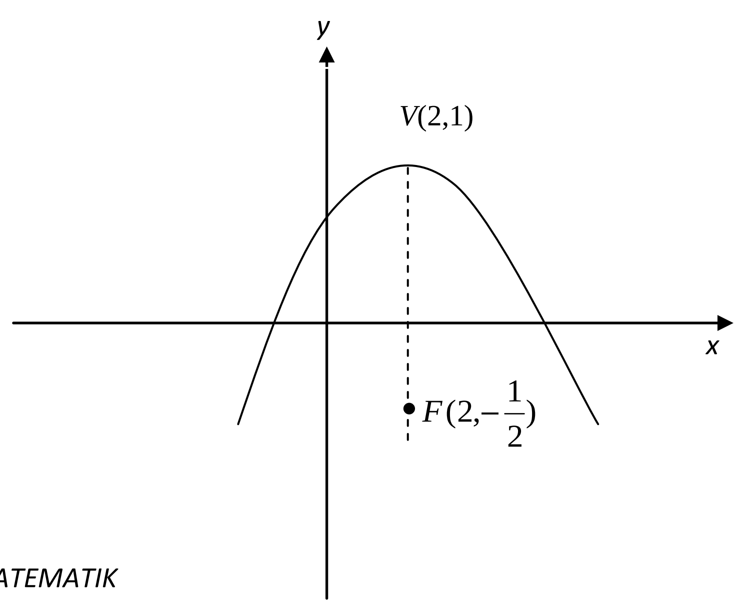
$(x-2)^2 - 4 + 6y - 2 = 0$

$(x-2)^2 = -6y + 6$

$(x-2)^2 = -6(y-1)$

$4a = -6$

$a = -\frac{3}{2}$



Vertex (2,1)

Focus $\left(2,-\frac{1}{2}\right)$

3. $x^2\frac{dy}{dx}+2xy=3x^2-1$

$\div x^2\ :\ \frac{dy}{dx}+\frac{2}{x}y=3-\frac{1}{x^2}$

$$\text{I}=e^{\int\frac{2}{x}dx}$$

$$=e^{2\ln x}\quad =x^2$$

$$x^2\frac{dy}{dx}+x^2\left(\frac{2}{x}y\right)=x^2\left(3-\frac{1}{x^2}\right)$$

$$x^2\frac{dy}{dx}+2xy=3x^2-1$$

$$\frac{d}{dx}(x^2y)=3x^2-1$$

$$\int\frac{d}{dx}(x^2y)=\int(3x^2-1)dx$$

$$x^2y=x^3-x+c$$

$when\quad x=1\quad and\quad y=2 \qquad\qquad \Rightarrow c=2$

$\therefore\qquad y=x-\frac{1}{x}+\frac{2}{x^2}$

4. a) Centre of circle = midpoint of AB

$$=\left(\frac{1+9}{2},\frac{2+0}{2}\right)=(5,1)$$

Radius = $\sqrt{(9-5)^2+(0-1)^2}$ = $\sqrt{17}$

$$(x-5)^2+(y-1)^2=\left(\sqrt{17}\right)^2$$
$$x^2-10x+25+y^2-2y+1-17=0$$
$$x^2+y^2-10x-2y+9=0$$

b) Equation of tangent :

$$xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$$

$(x_1, y_1) = (1,2)$

g = -5, f = -1 , c = 9

x (1) + y (2) – 5 (x +1) – 1(y +2) +9 =0

x + 2y – 5x – 5 – y – 2 + 9 = 0

y = 4x -2

5. $$\frac{1}{x(x^2+1)} = \frac{A}{x}+\frac{Bx+C}{\left(x^2+1\right)}$$

$$1 = A(x^2+1)+(Bx+C)x$$

$$1 = Ax^2+A+Bx^2+Cx$$

$$1 = (A+B)x^2+A+Cx$$

$$A=1 \qquad B=-1 \qquad C=0$$

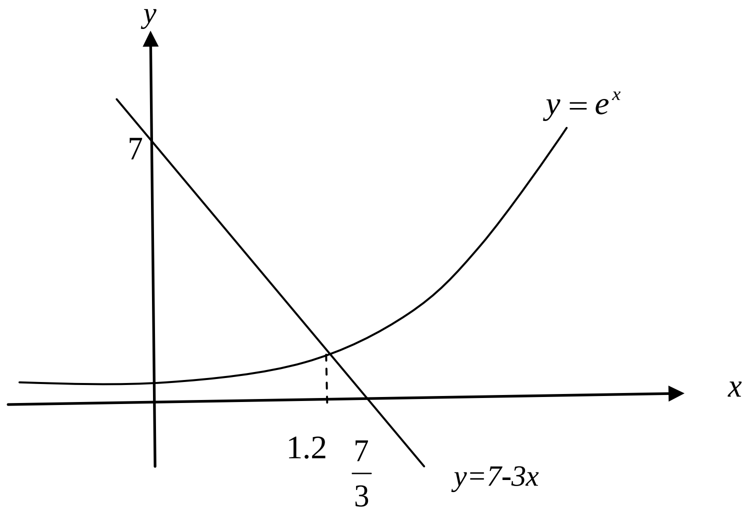
$$\therefore \frac{1}{x(x^2+1)} = \frac{1}{x}-\frac{x}{x^2+1}$$

$$\int_1^2 \frac{1}{x\left(x^2+1\right)}dx = \int_1^2 \frac{1}{x}dx - \int_1^2 \frac{x}{x^2+1}dx \qquad Let\ \ u=x^2+1 \qquad \frac{du}{2}=x\ \ dx$$

$$= \ln x-\frac{1}{2}\int\frac{1}{u}du = \left[\ln x-\frac{1}{2}\ln\left(x^2+1\right)\right]_1^2$$

$$= \left[\ln 2 - \frac{1}{2}\ln 5\right] - \left[\ln 1 - \frac{1}{2}\ln 2\right] = 0.2350$$

6. $y = e^x$, $y = 7 - 3x$



From the graph, $x_1 = 1.2$

$e^x + 3x - 7 = 0$

$f(x) = e^x + 3x - 7$

$f'(x) = e^x + 3$

$x_1 = 1.2$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 1.2 - \frac{e^{1.2} + 3(1.2) - 7}{e^{1.2} + 3}$$

$= 1.2126$

$$x_3 = 1.2126 - \frac{e^{1.2126} + 3(1.2126) - 7}{e^{1.2126} + 3}$$

$= 1.2126$

The root of the equation is 1.213