

1. Find the value of $\int_0^{\frac{\pi}{2}} \sin^2 x \cos^2 x dx$ [7 marks]
2. A conic section is given by the equation $x^2 - 4x + 6y - 2 = 0$. Express the equation of the conic section in the general form. Sketch the graph of the conic section and label the coordinates of its vertex and focus. [7 marks]
3. Solve the differential equation $x^2 \frac{dy}{dx} + 2xy = 3x^2 - 1$ for $y = 2$ when $x = 1$ [8 marks]
4. The end points of the diameter of a circle are A(1,2) and B(9,0). Determine:
- the equation of the circle.
 - the equation of the tangent line to the circle at point A . [8 marks]
5. Express $\frac{1}{x(x^2 + 1)} dx$ as a partial fractions. [5 marks]
- Hence find the value of $\int_1^2 \frac{1}{x(x^2 + 1)} dx$ [5 marks]
6. Sketch both the graphs $y = e^x$ and $y = 7 - 3x$ on the same coordinates axes. Use your graphs to obtain a first approximation to the root of the equation $e^x = 7 - 3x$. By using the Newton-Raphson method, determine the root of the equation correct to three decimal places. [10 marks]

END OF QUESTION

Final Answer

1. $\frac{\pi}{16}$

2. $(x-2)^2 = -6(y-1)$; vertex = (2,1) , focus = $\left(2, -\frac{1}{2}\right)$

3. $y = x - \frac{1}{x} + \frac{2}{x^2}$

4. a) $x^2 + y^2 - 10x - 2y + 9 = 0$

b) $y = 4x - 2$

5. $\frac{1}{x} - \frac{x}{x^2 + 1}$, 0.2350

6. 1.213

