1) $\int \frac{1}{1+e^{-2x}}\ dx = \int \frac{1}{1+\frac{1}{e^{2x}}}\ dx$

$u = e^{2x} + 1$

$du = 2e^{2x}\ dx$

$= \int \frac{e^{2x}}{e^{2x}+1}\ dx$

$= \int \frac{1}{u}\left(\frac{1}{2}\ du\right)$

$= \frac{1}{2}\ln\left|u\right| + C$

$= \frac{1}{2}\ln\left|e^{2x}+1\right| + C$

2) $a = 0,\ b = 2\ ,\ n = 4,\ y = f(x) = \frac{1}{1+x^2}$

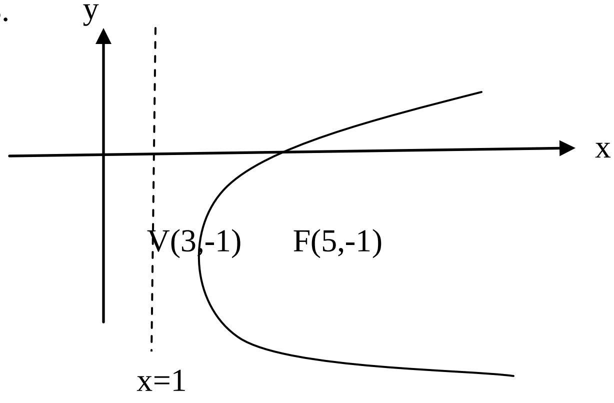
$h = \frac{b-a}{n} = \frac{2-0}{4} = 0.5$

x	$y = \frac{1}{1+x^2}$		
$x_0 = 0$	y_0	1	
$x_1 = 0.5$	y_1		0.8
$x_2 = 1.0$	y_2		0.5
$x_3 = 1.5$	y_3		0.30769
$x_4 = 2.0$	y_4	0.2	
Total		1.2	1.60769

$\int_0^2 \frac{1}{1+x^2}\ dx = \frac{0.5}{2}[1.2 + 2(1.60769)]$

$= 1.1038$

3.



$V(h,\ k) = V(3,\ -1)$
$h = 3,\ k = -1$
$F(h+p,\ k) = F(5,\ -1)$
$h + p = 5$
$3 + p = 5$
$p = 2$
$(y-k)^2 = 4p(x-h)$
$(y-(-1))^2 = 4(2)(x-3)$
$(y+1)^2 = 8(x-3)$

4. $\dfrac{dy}{dx} = 9x^2 y\sqrt{x^3-1}$

$u = x^3 - 1$
$du = 3x^2\ dx$

$\int \dfrac{1}{y}\ dy = \int 9x^2\sqrt{x^3-1}\ dx$

$\ln y = \int 9\sqrt{u}\ \left(\dfrac{1}{3}\ du\right)$

$= 3\int u^{\frac{1}{2}}\ du$

$= 3\left[\dfrac{u^{\frac{3}{2}}}{\frac{3}{2}}\right] + C$

$\ln y = 2(x^3-1)^{\frac{3}{2}} + C$

$y = Ae^{2(x^3-1)^{\frac{3}{2}}}$

$y(1) = 1 \Rightarrow x = 1,\ \ y = 1$

$1 = Ae^{2(1^3-1)^{\frac{3}{2}}}$

$A = 1$

$y = e^{2(x^3-1)^{\frac{3}{2}}}$

5. $x^2 + y^2 + 2gx + 2fy + c = 0$

At (1, -1) : $(1)^2 + (-1)^2 + 2g(1) + 2f(-1) + c = 0$

$2g - 2f + c = -2$ (1)

At (3, 5) : $(3)^2 + (5)^2 + 2g(3) + 2f(5) + c = 0$

$6g + 10f + c = -34$ (2)

$y = -\frac{2}{3}x + \frac{7}{3}$ passes through the center, so

$$-f = -\frac{2}{3}(-g) + \frac{7}{3}$$

$$f = -\frac{2}{3}g - \frac{7}{3} \qquad (3)$$

(2) – (1) : $4g + 12f = -32$

$$g + 3f = -8 \qquad (4)$$

Substituting (3) into (4) :

$$g + 3\left(-\frac{2}{3}g - \frac{7}{3}\right) = -8$$

$$-g - 7 = -8$$

$$g = 1$$

(3) : $f = -\frac{2}{3}(1) - \frac{7}{3}$

$$f = -3$$

(1) : $2(1) - 2(-3) + c = -2$

$$c = -10$$

$$x^2 + y^2 + 2(1)x + 2(-3)y + (-10) = 0$$

$$x^2 + y^2 + 2x - 6y - 10 = 0$$

6. $\int 8x^2e^{2x}\,dx = (8x^2)\left(\frac{e^{2x}}{2}\right) - \int\left(\frac{e^{2x}}{2}\right)(16x\,dx)$ $\qquad u = 8x^2 \Rightarrow du = 16x\,dx$, $\quad dv = e^{2x}\,dx \Rightarrow v = \frac{e^{2x}}{2}$

$$= 4x^2e^{2x} - \int 8xe^{2x}\,dx$$

$$= 4x^2e^{2x} - \left[(8x)\left(\frac{e^{2x}}{2}\right) - \int\left(\frac{e^{2x}}{2}\right)(8\,dx)\right]$$

$u = 8x \Rightarrow du = 8\,dx$, $\quad dv = e^{2x}\,dx \Rightarrow v = \frac{e^{2x}}{2}$

$$= 4x^2e^{2x} - \left[4xe^{2x} - \int 4e^{2x}\,dx\right]$$

$$= 4x^2e^{2x} - 4xe^{2x} + 4\left(\frac{e^{2x}}{2}\right) + C$$

$$= 2e^{2x}(2x^2 - 2x + 1) + C$$

Volume $= \pi\int_0^1 (5xe^x)^2\,dx$

$$= \pi\int_0^1 25x^2e^{2x}\,dx$$

$$= \frac{25}{8}\pi\int_0^1 8x^2e^{2x}\,dx$$

$$= \frac{25}{8}\pi\left[2e^{2x}(2x^2 - 2x + 1)\right]_0^1$$

$$= \frac{25\pi}{4}\left[e^2(2-2+1) - e^0(0-0+1)\right]$$

$$= \frac{25\pi}{4}(e^2 - 1)\ unit^3$$