1. $\int_3^4 \frac{4}{x^2-4}dx$

$$\frac{4}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$$

$$4 = A(x-2)(x+2)$$

Let $x=-2$, $4B = -4B \Rightarrow B = -1$

Let $x = 2$, $4 = 4B \Rightarrow A = 1$

$$\int_3^4 \frac{4}{x^2-4}dx = \int_3^4 \frac{1}{x-2} - \frac{1}{x+2}\,dx$$

$$= \left[\ln|x-2|\right]_3^4 - \left[\ln|x+2|\right]_3^4$$

$$= [\ln 2 - \ln 1] - [\ln 6 - \ln 5]$$

= ln2-ln6+ln5

$$= \ln\left(\frac{2x5}{6}\right)$$

$$= \ln\left(\frac{5}{3}\right) \text{ or } 0.51$$

2. $3\boldsymbol{x}^2 + 5\boldsymbol{y}^2 + 6\boldsymbol{x} - 20 = 0$

$$3\boldsymbol{x}^{2^2} + 6\boldsymbol{x} + 5\boldsymbol{y}^2 - 20 = 0$$

$$3(\boldsymbol{x}^2 + 2\boldsymbol{x}) + 5\boldsymbol{y}^2 - 20 = 0$$

$$3((\boldsymbol{x}+1)^2 - 1) + 5\boldsymbol{y}^2 = 20$$

$$3(\boldsymbol{x}+1)^2 + 5\boldsymbol{y}^2 = 23$$

$$3(\boldsymbol{x}+1)^2 + 5\boldsymbol{y}^2 = 23$$

$$\frac{3}{23}(\boldsymbol{x}+1)^2 + \frac{5}{23}\boldsymbol{y}^2 = \frac{23}{23}$$

$$\frac{(\boldsymbol{x}+1)^2}{23/3}+\frac{\boldsymbol{y}^2}{23/5}=1$$

$$\therefore \boldsymbol{C}(-1{,}0)$$

$$\boldsymbol{a}^2=\frac{23}{2}, \boldsymbol{b}^2=\frac{23}{5}$$

$$\boldsymbol{a}=\sqrt{\frac{23}{2}}, \boldsymbol{b}=\sqrt{\frac{23}{5}}$$

length of major axes $=2\sqrt{\dfrac{23}{2}}$

length of minor axes $=2\sqrt{\dfrac{23}{5}}$

3. a) Let $f(x)=x^3-4x+2$.

$$f(1)=1^3-4(1)+2=-1<0$$
$$f(2)=2^3-4(2)+2=2>0$$

$\therefore$ Since $f(1)$ and $f(2)$ have opposite signs and $f(x)$ is continuous function, therefore there is a root for $f(x)=0$ between $x=1$ and $x=2$.

b) Let $f(x)=x^3-4x+2$.

$$f'(x)=3x^2-4$$

$$x_\circ=2$$
$$x_1=x_\circ-\left[\frac{x_\circ{}^3-4x_\circ+2}{3x_\circ{}^2-4}\right]=1.75$$
$$x_2=1.6807$$
$$x_3=1.6752$$
$$x_4=1.6751$$
$$x_5=1.6751$$

$\therefore$ The root is 1.68.

4. a) $\frac{dy}{dx}+2y=xe^{-x}$

$V(x) = e^{\int 2dx}=e^{2x}$

$\int \frac{d}{dx}\left[ye^{2x}\right]dx=\int (xe^{x})\,dx$

$ye^{2x} = xe^{x}-\int e^{x}\,dx$

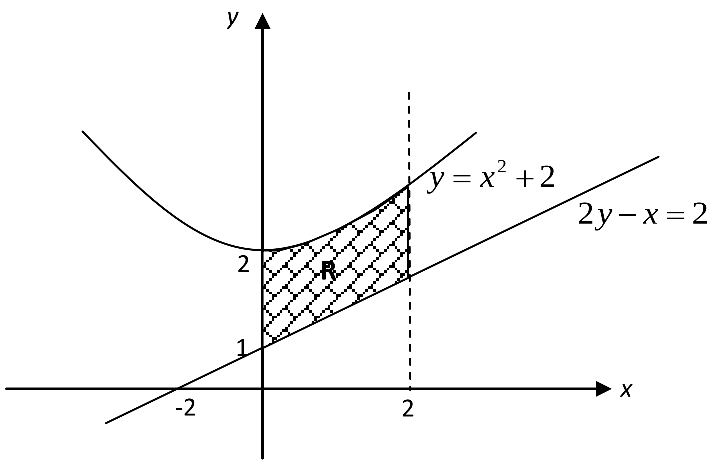
$ye^{2x} = xe^{x} - e^{x} + C$

When $x=0, y=3$, C=4

$ye^{2x} = xe^{x} - e^{x} + 4$

$y = \left(xe^{x} - e^{x} + 4\right)e^{-2x}$

5. a)



Area $= \int_0^2 \left[\left(x^2+2\right)-\left(\frac{x+2}{2}\right)\right]dx$

$= \frac{1}{2}\int_0^2 \left(2x^2-x+2\right)dx$

$$= \frac{1}{2}\left[\frac{2x^3}{3} - \frac{x^2}{2} + 2x\right]_0^2$$

$$= \frac{1}{2}\left[\left(\frac{16}{3} - 2 + 4\right) - 0\right]$$

$$= \frac{11}{3} \text{ unit}^2$$

(b) Volume $= \pi\int_0^2 \left[\left(x^2+2\right)^2 - \left(\frac{x+2}{2}\right)^2\right] dx$

$$= \frac{\pi}{4}\int_0^2 \left(4x^4 + 15x^2 - 4x + 12\right) dx$$

$$= \frac{\pi}{4}\left[\frac{4x^5}{5} + 5x^3 - 2x^2 + 12x\right]_0^2$$

$$= \frac{\pi}{4}\left[\left(\frac{128}{5} + 40 - 8 + 24\right) - 0\right]$$

$$= \frac{102}{5}\pi \text{ unit}^3$$

6. Radius $= \frac{|3+2|}{\sqrt{1^2+2^2}} = \frac{5}{\sqrt{5}}$ Equation of circle, $(x-3)^2 + (y-1)^2 = \left(\frac{5}{\sqrt{5}}\right)^2$

$$x^2 - 6x + 9 + y^2 - 2y + 1 = 5$$
$$x^2 + y^2 - 6x - 2y + 5 = 0$$

i) $x^2 + (mx)^2 - 6x - 2mx + 5 = 0$

$$x^2 + m^2x^2 - 6x - 2mx + 5 = 0$$
$$x^2(1+m^2) - (6+2m)x + 5 = 0$$
$$\therefore b^2 - 4ac = 0$$
$$(6+2m)^2 - 4(1+m^2)(5) = 0$$
$$4m^2 + 24m + 36 - 20 - 20m^2 = 0$$
$$2m^2 - 3m - 2 = 0$$
$$(2m+1)(m-2) = 0 \Rightarrow m = -\frac{1}{2} \, or \, 2$$

$\therefore y = 2x$ is another tangent from the origin.

ii) $y = 2x$ intersects $x^2 + y^2 - 6x - 2y + 5 = 0$

$$x^2 + (2x)^2 - 6x - 2(2x) + 5 = 0$$
$$x^2 - 2x + 1 = 0$$
$$(x-1)^2 = 0$$
$$x = 1 \Rightarrow y = 2$$

Point of intersection, $(1,2)$

Gradient of normal $= -\frac{1}{2}$

Equation of normal, $y - 2 = -\frac{1}{2}(x-1)$

$$2y - 4 = -x + 1$$
$$2y + x - 5 = 0$$