1. $\int_0^1 \left(x^2 - 1\right)\sqrt{x}\,dx$

$$= \int_0^1 x^{\frac{5}{2}} - x^{\frac{1}{2}}\,dx$$

$$= \left[\frac{2}{7}x^{\frac{7}{2}} - \frac{2}{3}x^{\frac{3}{2}}\right]_0^1$$

$$= \frac{2}{7} - \frac{2}{3} = -\frac{8}{21}$$

2. Separating the variables,

$$\frac{1}{\tan y}\,dy = \frac{1}{x^2}\,dx$$

$$= x^{-2}dx$$

Integrating,

$$\int \frac{1}{\tan y}\,dy = \int x^{-2}dx$$

$$\int \frac{\cos y}{\sin y}\,dy = \int x^{-2}dx$$

$\ln \sin y = -\frac{1}{x} + c$, where c is a constant……….(1)

$$\sin y = e^{-\frac{1}{x}+c}$$

$\sin y = Ae^{-\frac{1}{x}}$, where $A = e^{-c}$

Hence, the general solution is $\sin y = Ae^{-\frac{1}{x}}$.

3. $\int \frac{1}{1+e^{x}} dx$

Let $u = 1+e^{x}$

$du = e^{x} dx$

$\frac{du}{u-1} = dx$

Therefore,

$$\int \frac{1}{1+e^{x}} dx = \int \frac{1}{u} \cdot \frac{du}{u-1}$$

By partial fraction,

$$\frac{1}{u(u-1)} \equiv \frac{A}{u} + \frac{B}{u-1}$$

$$\frac{1}{u(u-1)} \equiv \frac{A(u-1)}{u} + \frac{Bu}{u-1}$$

$1 \equiv A(u-1) + Bu$

When $u = 0, A = -1$

When $u = 1, B = 1$

Therefore,

$$\int \frac{1}{1+e^{x}} dx = \int \frac{1}{u} \cdot \frac{du}{u-1}$$

$$= \int -\frac{1}{u} + \frac{1}{u-1} du$$

$$= -\ln|u| + \ln|u-1| + c$$

$$= \ln\left|\frac{u-1}{u}\right| + c$$

$$= \ln\left|\frac{e^x}{1+e^x}\right| + c$$

Alternative Method:

$$\int \frac{1}{1+e^x} dx$$

$$= \int \frac{1}{1+e^x} \frac{e^{-x}}{e^{-x}} dx$$

$$= \int \frac{e^{-x}}{e^{-x}+1} dx$$

$$= -\ln\left|e^{-x}+1\right| + c$$

4. Let $y = x^2 e^{2x}$, $n = 5$, $h = \frac{1-0}{5} = 0.2$

x	$y = x^2 e^{2x}$	
$x_0 = 0$	$y_0 = 0$	
$x_1 = 0.2$		$y_1 = 0.0597$
$x_2 = 0.4$		$y_2 = 0.3561$
$x_3 = 0.6$		$y_3 = 1.1952$
$x_4 = 0.8$		$y_4 = 3.1699$
$x_5 = 1$	$y_5 = 7.3891$	
Total	7.3891	4.7809

By the trapezoidal rule,

$$\int_0^1 x^2 e^{2x} dx \approx \frac{h}{2}\{y_0 + y_5 + 2(y_1 + y_2 + y_3 + y_4)\}$$

$$\approx \frac{0.2}{2}\{7.3891 + 2(4.7809)\}$$

$$\approx 1.6951$$

$\therefore \quad \int_0^1 x^2 e^{2x} dx \approx 1.695$ correct to three decimal places.

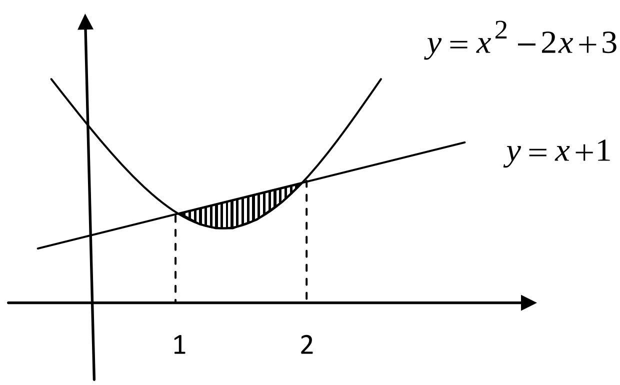
5. a) Solve simultaneously,

$y = x^2 - 2x + 3$

$y = x + 1$

The intersection points are $P(1, 2)$, $Q(2, 3)$

b)



c) volume $= \pi \int_1^2 (x+1)^2 - (x^2 - 2x + 3)^2 dx$

$$= \pi \int_1^2 -x^4 + 4x^3 - 9x^2 + 14x - 8 \ \ dx$$

$$= \pi \left[\frac{-x^5}{5} + x^4 - 3x^3 + 7x^2 - 8x \right]_1^2$$

$$= \pi\left(-\frac{12}{5} + \frac{16}{5}\right) = \frac{4}{5}\pi$$

6. $16x^2 + 4y^2 - 64x - 40y + 100 = 0$

$16(x^2 - 4x) + 4(y^2 - 10y) = -100$

$16(x^2 - 4x + 4) + 4(y^2 - 10y + 25) = 164 - 100$

that is, $16(x-2)^2 + 4(y-5)^2 = 64$

$$\frac{(x-2)^2}{4} + \frac{(y-5)^2}{16} = 1$$

This represents an ellipse with centre $(2,5)$.

Since the major axis is vertical, $a = 4$ and $b = 2$

$c^2 = a^2 - b^2$

$= 16 - 4 = 12$

$c = \sqrt{12}$

The foci are the points $(2, 5 + \sqrt{12})$, $(2, 5 - \sqrt{12})$.

The vertices the points $(2,9)$ and $(2,1)$.