1. Let $y = \dfrac{1}{\sin x}$, $n = 3$, $h = \dfrac{\frac{2\pi}{3} - \frac{\pi}{6}}{3} = \dfrac{\pi}{6}$

x	$y = \dfrac{1}{\sin x}$	
$x_0 = \dfrac{\pi}{6}$	$y_0 = 2$	
$x_1 = \dfrac{\pi}{3}$		$y_1 = 1.15470$
$x_2 = \dfrac{\pi}{2}$		$y_2 = 1$
$x_3 = \dfrac{2\pi}{3}$	$y_3 = 1.15470$	
Total	3.1547	2.1547

By the trapezoidal rule,

$$\int_{\frac{\pi}{6}}^{\frac{2\pi}{3}} \frac{1}{\sin x}\,dx \approx \frac{h}{2}\{y_0 + y_3 + 2(y_1 + y_2)\}$$

$$\approx \frac{1}{2}\left(\frac{\pi}{6}\right)\{3.1547 + 2(2.1547)\}$$

$$\approx 1.9540$$

$\therefore \displaystyle\int_{\frac{\pi}{6}}^{\frac{2\pi}{3}} \frac{1}{\sin x}\,dx = 1.954$ correct to three decimal places.

2. $x^2 + y^2 - 12x - 16y + 75 = 0$

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -12$ $2f = -16$ $c = 75$

$g = -6$ $f = -8$

Equation of tangent at point (x_1, y_1) is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

$\therefore$ The equation of tangent at point $(9,12)$ is

$$9x + 12y - 6(x + 9) - 8(y + 12) + 75 = 0$$

$$9x + 12y - 6x - 54 - 8y - 96 + 75 = 0$$

$$3x + 4y - 75 = 0$$

gradient of tangent, $m_1 = -\frac{3}{4}$

gradient of normal, $m_2 = \frac{4}{3}$

$\therefore$ The equation of normal at point $(9,12)$ is

$$y - 12 = \frac{4}{3}(x - 9)$$

$$3y - 36 = 4x - 36$$

$$3y - 4x = 0$$

3. $4x^2 - 8x + 9y^2 + 18y = 23$

$$4(x^2 - 2x) + 9(y^2 + 2y) - 23 = 0$$

$$4\left[(x-1)^2 - (-1)^2\right] + 9\left[(y+1)^2 - (1)^2\right] - 23 = 0$$

$$4(x-1)^2 - 4 + 9(y+1)^2 - 9 - 23 = 0$$

$$4(x-1)^2 + 9(y+1)^2 = 36$$

$$\frac{(x-1)^2}{9} + \frac{(y+1)^2}{4} = 1$$

This is an equation of an ellipse with centre $(1,-1)$.

$a^2 = 9 \quad \Rightarrow \quad a = \pm 3$

$b^2 = 4 \quad \Rightarrow \quad b = \pm 2$

$c = \sqrt{9-4} = \sqrt{5}$

Vertices : $(1 \pm 3, -1) \Rightarrow (-2,-1)$, $(4,-1)$

Foci : $(1 \pm \sqrt{5}, -1) \Rightarrow (1 - \sqrt{5}, -1)$, $(1 + \sqrt{5}, -1)$

4. a) $\int \sin x \cos^2 x \, dx$

Let $u=\cos x$

$$\frac{du}{dx}=-\sin x$$

$$-du=\sin x dx$$

$$\int \sin x\cos^2 x\ dx=-\int u^2 du$$

$$=-\frac{u^3}{3}+C$$

$$=-\frac{\cos^3 x}{3}+C$$

b) $\int \frac{x+14}{(x+4)(x-1)}dx = \int \frac{A}{x+4}+\frac{B}{x-1}dx$

$$\frac{x+14}{(x+4)(x-1)}=\frac{A(x-1)+B(x+4)}{(x+4)(x-1)}$$

$$x+14=A(x-1)+B(x+4)$$

when $x=1$; $15=5B \Rightarrow B=3$

when $x=-4$; $10=-5A \Rightarrow A=-2$

$$\int \frac{x+14}{(x+4)(x-1)}dx=\int \frac{-2}{x+4}+\frac{3}{x-1}dx$$

$$=-2\ln|x+4|+3\ln|x-1|+C$$

$$=\ln\left|\frac{(x-1)^3}{(x+4)^2}\right|+C$$

5. a) $\frac{dy}{dx}-\frac{x^2+4}{3y}=0$

$$\frac{dy}{dx}=\frac{x^2+4}{3y}$$

$$\int 3y dy=\int (x^2+4)dx$$

$$\frac{3}{2}y^2=\frac{x^3}{3}+4x+C$$

$$y^2=\frac{2}{3}\left(\frac{x^3}{3}+4x\right)+C$$

b) $\frac{dy}{dx} - \frac{2y}{x} = 3x^2$

$P(x) = -\frac{2}{x}$; $Q(x) = 3x^2$

$V(x) = e^{-\int \frac{2}{x}dx}$

$= e^{-2\ln x}$

$= e^{\ln x^{-2}}$

$= \frac{1}{x^2}$

$\frac{1}{x^2}\left(\frac{dy}{dx} - \frac{2y}{x}\right) = 3x^2\left(\frac{1}{x^2}\right)$

$\frac{d}{dx}\left(\frac{y}{x^2}\right) = 3$

$\int \frac{d}{dx}\left(\frac{y}{x^2}\right)dx = \int 3dx$

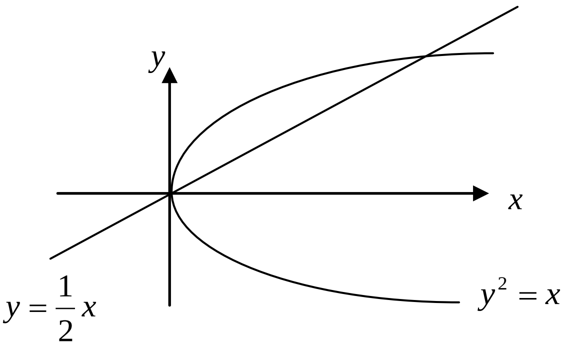
$\frac{y}{x^2} = 3x + C$

when $x = 1,\ y = 4$

$4 = 3 + C \quad \Rightarrow \quad C = 1$

$\therefore\ y = 3x^3 + x^2$

6.



$y = \frac{1}{2}x$ ----- (1)

$y^2 = x$ ----- (2)

substitute (1) into (2):

$x^2 = 4x$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0, x = 4$

when $x = 0$, $y = 0$

when $x = 4$, $y = \frac{1}{2}(4) = 2$

$\therefore$ The points of intersections are (0, 0) and (4, 2).

$$\text{Volume} = \pi\int_0^4 \left(\sqrt{x}\right)^2 - \left(\frac{1}{2}x\right)^2 dx$$

$$= \pi\int_0^4 x - \frac{1}{4}x^2 dx$$

$$= \pi\left[\frac{x^2}{2} - \frac{x^3}{12}\right]_0^4$$

$$= \pi\left[\frac{16}{2} - \frac{64}{12}\right] - 0$$

$$= 2\frac{2}{3}\pi\ unit^3$$