1. Use the Trapezoidal rule with 3 sub-intervals to estimate $\int_{\frac{\pi}{6}}^{\frac{2\pi}{3}} \frac{1}{\sin x}\,dx$ correct to 3 decimal places. **[5 marks]**

2. Find the equations of the tangent and normal to the circle $x^2 + y^2 - 12x - 16y + 75 = 0$ at the point $(9,12)$. **[6 marks]**

3. Show that the equation $4x^2 - 8x + 9y^2 + 18y = 23$ represents an ellipse. Hence, find the centre of the ellipse and its foci. **[9 marks]**

4. Find the following integrals :

   a) $\int \sin x \cos^2 x \; dx$ **[4 marks]**

   b) $\int \frac{x+14}{x^2+3x-4}\,dx$. **[6 marks]**

5. a) Solve $\frac{dy}{dx} - \frac{x^2+4}{3y} = 0$. **[4 marks]**

   b) If $\frac{dy}{dx} - \frac{2y}{x} = 3x^2$ and $y = 4$ when $x = 1$, find y in terms of x. **[6 marks]**

6. Sketch, on the same coordinate axes, the line $y = \frac{1}{2}x$ and the curve $y^2 = x$. Find the coordinates of the point of intersection. Find the volume formed by the rotation through 2π radians about x-axis of the region bounded by the line $y = \frac{1}{2}x$ and the curve $y^2 = x$.

   [10 marks]

END OF QUESTION

Final Answer

1. 1.954

2. $3x+4y-75=0$; $3y-4x=0$

3. Centre : $(1,-1)$

   Vertices : $(-2,-1)$, $(4,-1)$

   Foci : $(1-\sqrt{5},-1)$, $(1+\sqrt{5},-1)$

4. a) $-\dfrac{\cos^3 x}{3}+C$

   b) $\ln\left|\dfrac{(x-1)^3}{(x+4)^2}\right|+C$

5. a) $y^2=\dfrac{2}{3}\left(\dfrac{x^3}{3}+4x\right)+C$

   b) $y=3x^3+x^2$

6. Volume = $2\dfrac{2}{3}\pi\ unit^3$