

$$1. (s^2 + 1) \frac{dr}{ds} = rs$$

$$\int \frac{1}{r} dr = \int \frac{s}{s^2 + 1} ds$$

$$\ln|r| = \frac{1}{2} \ln|s^2 + 1| + c$$

$$r = 1, s = 2$$

$$0 = \frac{1}{2} \ln 5 + c$$

$$\ln|r| = \frac{1}{2} \ln|s^2 + 1| - \frac{1}{2} \ln 5$$

$$2 \ln|r| = \ln \left| \frac{s^2 + 1}{5} \right|$$

$$r^2 = \frac{s^2 + 1}{5}$$

$$5r^2 = s^2 + 1$$

$$2. \text{ Let } x = \sqrt{6}$$

$$x^2 = 6$$

$$x^2 - 6 = 0$$

$$f(x) = x^2 - 6$$

$$f'(x) = 2x$$

$$x_n = x - \frac{x^2 - 6}{2x}$$

$$x_0 = 2.5$$

$$x_1 = 2.5 - \frac{2.5^2 - 6}{2(2.5)} = 2.45$$

$$x_2 = 2.45 - \frac{2.45^2 - 6}{2(2.45)} = 2.44949$$

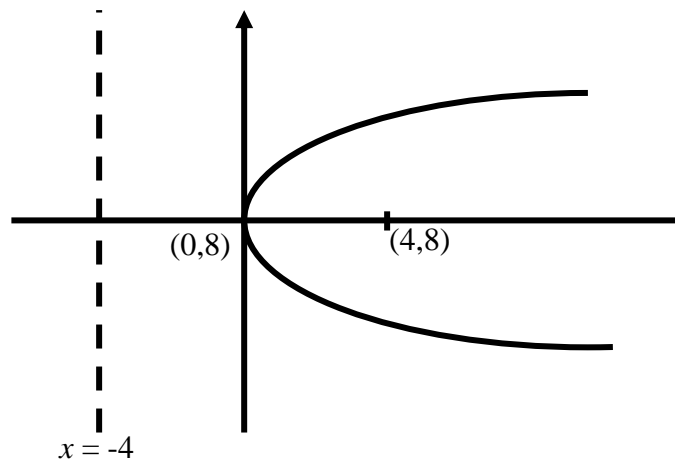
$$x_3 = 2.44949 - \frac{2.44949^2 - 6}{2(2.44949)} = 2.44949$$

$$x_4 = 2.44949 - \frac{2.44949^2 - 6}{2(2.44949)} = 2.44949$$

$$\therefore x = 2.449499$$

3. $(y - 8)^2 = 4(4)x$

$$(y - 8)^2 = 16x$$



4. a) $\int \frac{1}{x^2} (4 - 3x^4) dx$

$$= \int 4x^{-2} - 3x^2 dx$$

$$= \frac{4x^{-1}}{-1} - \frac{3x^3}{3} + c$$

$$= -\frac{4}{x} - x^3 + c$$

b) let $u = x - 1 \Rightarrow x = u + 1$

$$du = dx$$

$$\int_2^3 \frac{x}{(x-1)^2} dx = \int_1^2 \frac{u+1}{u^2} du$$

$$\begin{aligned}
 &= \int_1^2 \frac{1}{u} + \frac{1}{u^2} du \\
 &= \left[\ln u - \frac{1}{u} \right]_1^2 \quad \text{or} \quad \left[\ln(x-1) - \frac{1}{x-1} \right]_2^3 \\
 &= (\ln 2 - \ln 1) - \left(\frac{1}{2} - 1 \right) \\
 &= \ln 2 + \frac{1}{2}
 \end{aligned}$$

5. $x^2 + y^2 - x - \frac{5}{2}y + \frac{3}{2} = 0$

$$\left(x - \frac{1}{2}\right)^2 - \frac{1}{4} + \left(y - \frac{5}{4}\right)^2 - \frac{25}{16} = -\frac{3}{2}$$

$$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{5}{4}\right)^2 = \frac{5}{16}$$

$$\text{centre} = \left(\frac{1}{2}, \frac{5}{4}\right) \quad r = \sqrt{\frac{5}{16}}$$

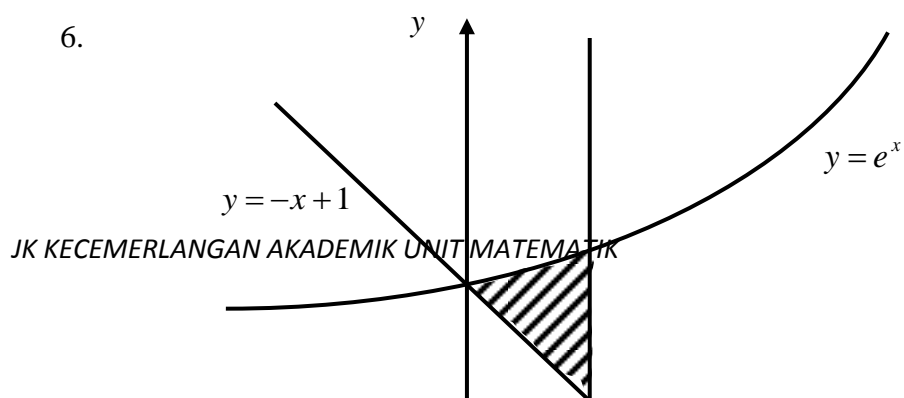
$$M_N = \frac{\frac{5}{4} - 1}{\frac{1}{2} - 1} = -\frac{1}{2} \qquad M_T = 2$$

Equation normal: $y - 1 = -\frac{1}{2}(x - 1)$

$$2y + x - 3 = 0$$

Equation tangent is: $y - 1 = 2(x - 1)$

6.



$$\begin{aligned}\text{Area} &= \int_0^1 e^x - (-x+1) dx \\ &= \left| e^x + \frac{x^2}{2} - x \right|_0^1 \\ &= \left(e + \frac{1}{2} - 1 \right) - (1) \\ &= e - \frac{3}{2} \text{ unit}^2\end{aligned}$$

$$\begin{aligned}\text{Volume} &= \pi \int_0^1 (e^x)^2 - (-x+1)^2 dx \\ &= \pi \left| \frac{e^{2x}}{2} - \frac{x^3}{3} + x^2 - x \right|_0^1 \\ &= \pi \left[\left(\frac{e^2}{2} - \frac{1}{3} + 1 - 1 \right) - \left(\frac{1}{2} \right) \right] \\ &= \left(\frac{e^2}{2} - \frac{5}{6} \right) \pi \text{ unit}^3\end{aligned}$$