1. $4x^2 - 8x + y^2 + 4y + 4 = 0$

$4(x^2 - 2x) + y^2 + 4y + 4 = 0$

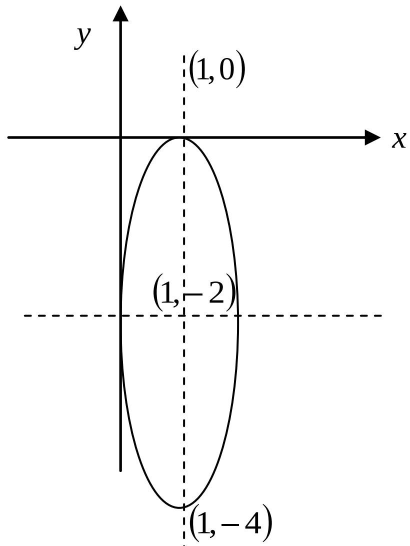
$4\left[x^2 - 2x + \left(-\frac{2}{2}\right)^2 - \left(-\frac{2}{2}\right)^2\right] + \left[y^2 + 4y + \left(\frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2\right] + 4 = 0$

$4(x-1)^2 + (y+2)^2 = 4$

$\frac{(x-1)^2}{1} + \frac{(y+2)^2}{4} = 1$

Centre : $(1, -2)$

Vertices : $(1, 0)$ and $(1, -4)$



2. $h(x) = |x-2| - 3$

a) $h(0) = |0-2| - 3 = -1$

$h(2) = -3$

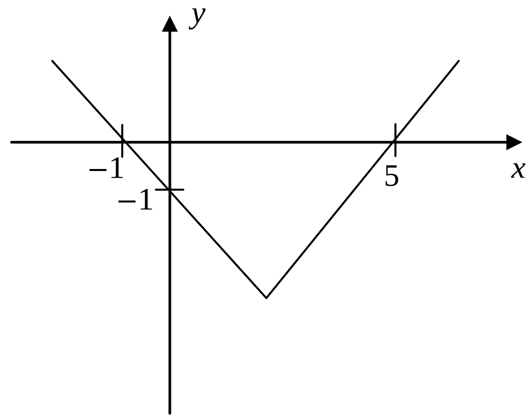
When $|x-2| - 3 = 0$

$x - 2 = \pm 3$

$x = -1 \text{ or } 5$

$D_h : (-\infty, \infty)$

$R_h : [-3, \infty)$



b) $\int_{-6}^{6} h(x)dx = \int_{-6}^{2} [-(x-2)-3]dx + \int_{2}^{6} (x-2-3)dx$

$$= \left[\frac{-x^2}{2} - x\right]_{-6}^{2} + \left[\frac{x^2}{2} - 5x\right]_{2}^{6}$$

$$= (-2-2)-(-18+6)+(18-30)-(2-10)$$

$$= 4$$

3. $\frac{dy}{dx} + 2y = xe^{-x}$

$$e^{\int 2dx} = e^{2x}$$

$$e^{2x}\frac{dy}{dx} + 2ye^{2x} = xe^{-x}e^{2x}$$

$$\frac{d}{dx}(ye^{2x}) = xe^{x}$$

$$ye^{2x} = \int xe^{x}dx$$

$$ye^{2x} = xe^{x} - \int e^{x}dx$$

$$ye^{2x} = xe^{x} - e^{x} + c$$

When $x = 0$, $y = 3$, $3e^{0} = 0 - e^{0} + c$

$$c = 4$$

$$ye^{2x} = xe^{x} - e^{x} + 4$$

$$y = (xe^{x} - e^{x} + 4)e^{-2x}$$

4. Let $f(x) = 2x^3 + x^2 - 2$

$$f(0.5) = -1.5$$

$f(1) = 1$

Sign changes, there's a root between [0.5,1]

$f'(x) = 6x^2 + 2x$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Let $x_0 = 0.75$

$$x_1 = 0.75 - \frac{f(0.75)}{f'(0.75)}$$

$$x_1 = 0.75 - \frac{\left[2(0.75)^3 + (0.75)^2 - 2\right]}{6(0.75)^2 + 2(0.75)}$$

$x_1 = 0.871795$

$x_2 = 0.858279$

$x_3 = 0.8580943$

$x_4 = 0.8580943$

$\therefore$ The root is 0.8581 (4 d.p)

5. a) Let $u = e^{2x} + 1$

$$\frac{du}{dx} = 2e^{2x}$$

$$dx = \frac{du}{2e^{2x}}$$

$$\int \frac{e^{2x}}{e^{2x}+1}\,dx = \int \frac{1}{2}\left(\frac{1}{u}\right)du$$

$$= \frac{1}{2}\int \frac{1}{u}\,du$$

$$= \frac{1}{2}\ln|u| + c$$

$$= \frac{1}{2}\ln\left|e^{2x} + 1\right| + c$$

b) Let $u = x$ and $dv = \cos 2x$

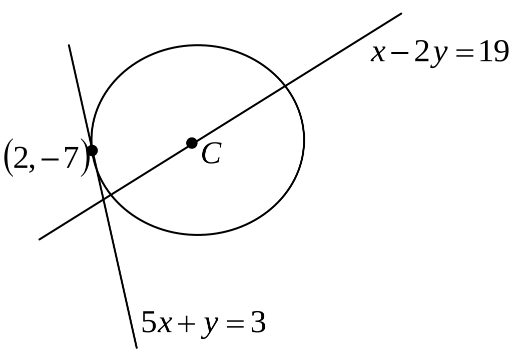
$du = dx$ $v = \dfrac{\sin 2x}{2}$

$$\int u\,dv = uv - \int v\,du$$

$$\int x\cos 2x = x\left(\frac{\sin 2x}{2}\right) - \int \frac{\sin 2x}{2}\,dx$$

$$= \frac{x\sin 2x}{2} + \frac{1}{4}\cos 2x + c$$

6.



The gradient of the line $5x + y = 3$ is $m_1 = -5$

The gradient of normal is $m_2 = \dfrac{1}{5}$

$$y - (-7) = \frac{1}{5}(x - 2)$$

$5y + 35 = x - 2$

$x - 5y = 37$ ....(1)

$x - 2y = 19$ ....(2)

(2)-(1) : $3y = -18$

$y = -6$; $x = 7$

Point of intersection is $(7, -6)$

Centre $(7, -6)$

The radius, $r = \sqrt{(7-2)^2 + (-6+7)^2}$

$= \sqrt{26}$

$\therefore$ The equation of the circle is $(x-7)^2 + (y+6)^2 = 26$