

1. Express the equation of the ellipse $4x^2 + y^2 - 8x + 4y + 4 = 0$ in the standard form. Determine the coordinates of the centre and the vertices on the major axis. Hence sketch the graph. [7 marks]
2. Given $h(x) = |x - 2| - 3$
- a) Sketch the graph of $h(x)$ [2 marks]
- b) Find $\int_{-6}^6 h(x) dx$ [5 marks]
3. Solve the differential equation $\frac{dy}{dx} + 2y = xe^{-x}$ given that $y = 3$ when $x = 0$ [7 marks]
4. Show that $2x^3 + x^2 - 2 = 0$ has a root in $[0.5, 1]$. Use Newton-Raphson method to find this root, correct to 4 decimal places. [7 marks]
5. Find
- a) $\int \frac{e^{2x}}{e^{2x} + 1} dx$ [4 marks]
- b) $\int x \cos 2x dx$ [6 marks]
6. A circle touches the line $5x + y = 3$ at the point $(2, -7)$ and its centre lies on the line $x - 2y = 19$. Find the point of intersection between the normal to the circle at $(2, -7)$ and the line $x - 2y = 19$. Hence, determine the centre and the standard equation of the circle. [12 marks]

END OF QUESTION

Final Answer

1. $\frac{(x-1)^2}{1} + \frac{(y+2)^2}{4} = 1, C(1,-2), V_1(1,0), V_2(1,-4)$
2. b) 4
3. $(xe^x - e^x + 4)e^{-2x}$
4. 0.8581
5. a) $\frac{1}{2} \ln|e^{2x} + 1| + c$
b) $\frac{x}{2} \sin 2x + \frac{1}{4} \cos 2x + c$
6. Point of intersection is (7,-6), Centre (7,-6), $(x-7)^2 + (y+6)^2 = 26$.

