QS016/2
Mathematics
Paper 2
Semester I
Session 2010/2011
2 hours

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Matematik
Kertas 2
Semester I
Sesi 2010/2011
2 jam

BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 2
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **15** halaman bercetak.
This booklet consists of **15** *printed pages.*

SHAMMAL

INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Differentiation

If $y = g(t)$ and $x = f(t)$, then $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

Integration

$$\int u\,dv = uv - \int v\,du$$

1 Find $\dfrac{dy}{dx}$ for each of the following:

(a) $y = (\ln x)^5$.

[2 *marks*]

(b) $xy^2 - ye^x = 3$.

[4 *marks*]

2 Find the exact value of $\displaystyle\int_1^{\sqrt{2}} t^3\sqrt{t^2 - 1}\, dt$.

[6 *marks*]

3 If f is a function with $f'(1) = 2$, find $\displaystyle\lim_{x \to 1} \frac{f(x) - f(1)}{\sqrt{x} - 1}$.

[6 *marks*]

4 Express $\dfrac{8x^2 + 15}{2x^3 + 3x}$ as partial fractions.

Hence, evaluate $\displaystyle\int \frac{8x^2 + 15}{2x^3 + 3x}\, dx$.

[7 *marks*]

5 Given the functions f and g as follows:

$$f(x) = 2 - x^2,$$
$$g(x) = x + 2.$$

(a) Find $f \circ g$ and $g \circ f$.

[4 *marks*]

(b) State the domain and range of $f \circ g$.

[3 *marks*]

(c) Find $(g \circ f)^{-1}$.

[2 *marks*]

(d) Determine the value of x such that $f \circ g(x) = g \circ f(x)$.

[3 *marks*]

6 (a) State the conditions of continuity of a function at a point $x = c$.

[2 *marks*]

(b) A function f defined by

$$f(x) = \begin{cases} \dfrac{|x-2|}{x^2+3x-10} & , \quad -5 < x < 2 \\ A & , \quad 2 \le x < 3 \\ Ax + B & , \quad x = 3 \end{cases}$$

is continuous at $x = 2$ and $x = 3$.

(i) Find $\lim\limits_{x \to 2^-} f(x)$.

[6 *marks*]

(ii) Determine the values of the constants A and B.

[5 *marks*]

7 (a) Evaluate.

(i) $\lim_{x \to \infty} \frac{\sqrt{4x^2 + 2x - 1}}{x + 1}$.

[3 *marks*]

(ii) $\lim_{x \to -3} \frac{2 - \sqrt{x^2 - 5}}{x + 3}$.

[4 *marks*]

(b) If $\lim_{x \to 4} \frac{f(x) - 5}{x - 2} = 1$, find $\lim_{x \to 4} f(x)$.

[3 *marks*]

8 Consider the curve given by the equation $f(x) = 2 - x^2$.

(a) Sketch the region bounded by the curves $f(x)$, $g(x) = x^2$, the lines $x = 0$ and $x = 2$. Hence, find the area of the region.

[7 *marks*]

(b) Find the volume of solid generated when the region bounded by the curve $f(x)$, lines $x = 1$ and $x = 2$ is rotated completely about the x – axis.

[5 *marks*]

9 Consider the parametric equations

$$x = 2t - t^{-1}, \qquad y = 2t + t^{-1}, \qquad t \geq 1.$$

(a) Show that

$$\frac{dy}{dx} = \frac{2t^2 - 1}{2t^2 + 1}.$$

[3 *marks*]

(b) Evaluate $\frac{dy}{dx}$ at the point $(1, 3)$.

[4 *marks*]

(c) Find $\frac{d^2y}{dx^2}$ in term of t. Hence, show that

$$\frac{d^2y}{dx^2} = \frac{8}{y^3}.$$

[6 *marks*]

10 A function f is defined by $f(x) = \dfrac{5x^2 + 8x + 4}{x^2 + x}$.

(a) Find the vertical and horizontal asymptotes of f.

[3 *marks*]

(b) Find the coordinates of the point where the curve f cuts the horizontal asymptote.

[2 *marks*]

(c) Determine the coordinates of the point where $f'(x) = 0$.

[3 *marks*]

(d) By writing $y = f(x)$, show that

$$(y-5)x^2 + (y-8)x - 4 = 0$$

Hence, for real x, show that $f(x) \leq -4$ or $f(x) \geq 4$.

[4 *marks*]

(e) Sketch the graph of f.

[3 *marks*]

END OF QUESTION BOOKLET