QS015/2
Mathematics
Paper 2
Semester I
Session 2013/2014
2 hours

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Matematik
Kertas 2
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2 jam

BAHAGIAN MATRIKULASI
KEMENTERIAN PENDIDIKAN MALAYSIA
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PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 2
2 jam

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SHAMMAL

1 Express $\frac{x^2}{x^2+3x+2}$ in partial fractions form.

[5 *marks*]

2 State the values of R and α such that $3\sin\theta+6\cos\theta = R\sin(\theta+\alpha)$ where $R>0$ and $0^\circ < \alpha \leq 90^\circ$. Hence, solve $3\sin\theta+6\cos\theta=\sqrt{5}$ for $0^\circ \leq \theta < 180^\circ$.

[6 *marks*]

3 (a) Find the value of m if $\lim\limits_{x\to 0}\frac{mx+3x^2}{4x-8x^2}=3$.

[3 *marks*]

(b) Evaluate $\lim\limits_{x\to 0}\frac{\sqrt{3-x}-\sqrt{3}}{x}$.

[4 *marks*]

4 (a) Find $\frac{dy}{dx}$ if $y=\operatorname{cosec}\left\{\sin\left[\ln(x+1)\right]\right\}$.

[3 *marks*]

(b) Obtain the second derivative of $y=\frac{\cos 3x}{e^{2x}}$ and express your answer in the simplest form.

[4 *marks*]

5 A cubic polynomial $P(x)$ has remainders 3 and 1 when divided by $(x-1)$ and $(x-2)$, respectively.

(a) Let $Q(x)$ be a linear factor such that $P(x)=(x-1)(x-2)Q(x)+\alpha x+\beta$, where α and β are constants. Find the remainder when $P(x)$ is divided by $(x-1)(x-2)$.

[5 *marks*]

(b) Use the values of α and β from part (a) to determine $Q(x)$ if the coefficient of x^3 for $P(x)$ is 1 and $P(3)=7$. Hence, solve for x if $P(x)=7-3x$.

[6 *marks*]

6 (a) State the definition of the continuity of a function at a point. Hence, find the value of d such that

$$f(x)=\begin{cases} e^{3x+d}, & x\le 0 \\ 3x+5, & x>0 \end{cases}$$

is continuous at $x=0$.

[5 *marks*]

(b) A function f is defined by

$$f(x)=\begin{cases} x^2-1, & x\le 1 \\ k(x-1), & x>1. \end{cases}$$

Determine the value(s) of k if f is:

(i) continuous for all $x\in\mathbb{R}$.

[3 *marks*]

(ii) differentiable for all $x\in\mathbb{R}$.

[4 *marks*]

7 (a) Find the derivative of $f(x)=\frac{1}{x+1}$ by using the first principle.

[4 *marks*]

(b) Use implicit differentiation to find:

(i) $\frac{dy}{dx}$ if $y\ln x = e^{x-y}$.

[3 *marks*]

(ii) the value of $\frac{dy}{dx}$ if $\frac{1}{y}-\frac{1}{x}=3$ when $x=\frac{1}{2}$.

[5 *marks*]

8 A curve is defined by parametric equations

$x=\ln(1+t),\ y=e^{t^2}$ for $t>-1$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of t.

[6 *marks*]

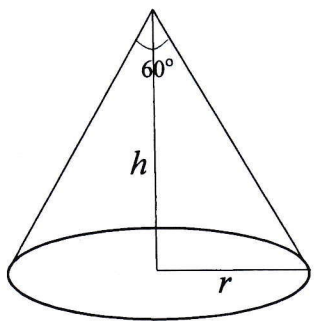
(b) Show that the curve has only one relative extremum at $(0,1)$ and determine the nature of the point.

[6 *marks*]

9 (a) A cylindrical container of volume 128π m^3 is to be constructed with the same material for the top, bottom and lateral side. Find the dimensions of the container that will minimise the amount of the material needed.

[6 *marks*]

(b) Gravel is poured onto a flat ground at the rate of $\frac{3}{20}$ m^3 per minute to form a conical-shaped pile with vertex angle $60°$ as shown in the diagram below.



Compute the rate of change of the height of the conical pile at the instant $t = 10$ minutes.

[7 *marks*]

10 (a) Show that $\dfrac{\sin\alpha+\sin\beta}{\cos\alpha-\cos\beta}=\cot\left(\dfrac{\beta-\alpha}{2}\right)$.

[4 *marks*]

(b) Use trigonometric identities to verify that

(i) $\sin\theta=\dfrac{2\tan\dfrac{\theta}{2}}{1+\tan^2\dfrac{\theta}{2}}$.

[3 *marks*]

(ii) $\cos\theta=\dfrac{1-\tan^2\dfrac{\theta}{2}}{1+\tan^2\dfrac{\theta}{2}}$.

[3 *marks*]

Hence, solve the equation $3\sin\theta+\cos\theta=2$ for $0^\circ\le\theta\le180^\circ$. Give your answers correct to three decimal places.

[5 *marks*]

END OF QUESTION PAPER