QS015/2 Mathematics Paper 2 Semester I Session 2011/2012 2 hours QS015/2 Matematik Kertas 2 Semester I Sesi 2011/2012 2 jam



BAHAGIAN MATRIKULASI KEMENTERIAN PELAJARAN MALAYSIA

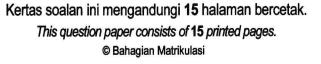
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI

MATRICULATION PROGRAMME EXAMINATION

MATEMATIK Kertas 2 2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU. DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.



SHAMMAL

1 Express $\frac{6x-13}{(3x-4)^2}$ in the form of partial fractions.

[5 marks]

2 Evaluate the following limits:

(a)
$$\lim_{x\to 2} \frac{x^4-16}{x-2}$$
.

[3 marks]

(b)
$$\lim_{x \to +\infty} \frac{\sqrt{x+1} + \sqrt{x}}{\sqrt{x}}.$$

[3 marks]

3 Find $\frac{dy}{dx}$ for the following equations:

(a)
$$y = 3^{2x+1}$$
.

[3 marks]

(b)
$$e^{xy} + y = 5x$$
.

[3 marks]

The surface area of a balloon in the shape of a sphere is decreasing at the rate of $2 \text{ cm}^2/\text{min}$. Find the rate at which the volume is decreasing when the radius of the balloon is 5 cm.

[7 marks]

5 (a) The function $f(x) = x^3 - 6x^2 + 9x - 3$ is defined on the interval [0, 5]. Find the critical points of f(x) on this interval and determine whether the critical points are local minimum or maximum.

[6 marks]

(b) Find the horizontal and vertical asymptotes for $f(x) = \frac{3x}{\sqrt{x^2 - 16}}$.

[7 marks]

- The polynomial $p(x) = x^3 2x^2 + ax + b$, where a and b are constants, has a factor of (x-2) and leaves a remainder of a^3 when it is divided by (x-a).
 - (a) Find the values of a and b.

[6 marks]

(b) Factorize p(x) completely by using the values of a and b obtained from part b(a). Hence, find the real roots of p(x) = 0, where a and b are not equal to zero.

[6 marks]

- Given that $x = \frac{1}{\sqrt{1+t^2}}$ and $y = \frac{\sqrt{1+t^2}}{t}$, where t is a non zero parameter.
 - (a) Show that $\frac{dy}{dx} = \frac{1+t^2}{t^3}$.

[6 marks]

(b) Find $\frac{d^2y}{dx^2}$ when t=1.

[6 marks]

8 (a) If $y = \sin(x^2 + 1)$, show that

$$x\frac{d^2y}{dx^2} - \frac{dy}{dx} + 4x^3y = 0.$$

[5 marks]

(b) Find the gradient of a curve $xe^{xy} = e^{2x} - e^{3y}$ at (0, 0).

[6 marks]

- 9 (a) Given $f(x) = \begin{cases} \frac{x^3 64}{x 4}, & x \neq 4 \\ 40, & x = 4. \end{cases}$
 - (i) Find $\lim_{x\to 4} f(x)$.

[4 marks]

(ii) Is f continuous at x = 4? Give your reason.

[3 marks]

(b) Determine the values of A and B such that the function

$$h(x) = \begin{cases} Ax - B, & x \le -1 \\ 2x^2 + 3Ax + B, & -1 < x \le 1 \\ 4, & x > 1. \end{cases}$$

is continuous for all values of x.

[6 marks]

10 (a) Given $\tan \frac{\pi}{3} = \sqrt{3}$ and $\tan \frac{\pi}{4} = 1$.

Express $\tan \frac{7\pi}{12}$ in the form of $a + \sqrt{b}$ where a and b are integers.

Hence, show that $\tan\left(\frac{7\pi}{6}\right) = \frac{1}{\sqrt{b}}$.

[6 marks]

(b) Find R and α such that the expression $9\sin\theta + 12\cos\theta$ can be expressed in the form of $R\sin(\theta + \alpha)$, where R > 0, $0^{\circ} < \alpha < 90^{\circ}$.

Hence, if $9\sin\theta + 12\cos\theta = 5$, solve for θ in the interval $0^{\circ} \le \theta \le 360^{\circ}$.

[9 marks]

END OF QUESTION PAPER