

QS015/2
Mathematics
Paper 2
Semester I
Session 2011/2012
2 hours

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Matematik
Kertas 2
Semester I
Sesi 2011/2012
2 jam



BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
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PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK

Kertas 2

2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **15** halaman bercetak.

This question paper consists of 15 printed pages.

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SHAMMIL

- 1 Express $\frac{6x-13}{(3x-4)^2}$ in the form of partial fractions.

[5 marks]

- 2 Evaluate the following limits:

(a) $\lim_{x \rightarrow 2} \frac{x^4 - 16}{x - 2}.$

[3 marks]

(b) $\lim_{x \rightarrow +\infty} \frac{\sqrt{x+1} + \sqrt{x}}{\sqrt{x}}.$

[3 marks]

- 3 Find $\frac{dy}{dx}$ for the following equations:

(a) $y = 3^{2x+1}.$

[3 marks]

(b) $e^{xy} + y = 5x.$

[3 marks]

- 4 The surface area of a balloon in the shape of a sphere is decreasing at the rate of $2 \text{ cm}^2/\text{min}$. Find the rate at which the volume is decreasing when the radius of the balloon is 5 cm.

[7 marks]

- 5 (a) The function $f(x) = x^3 - 6x^2 + 9x - 3$ is defined on the interval $[0, 5]$.
Find the critical points of $f(x)$ on this interval and determine whether the critical points are local minimum or maximum.
[6 marks]
- (b) Find the horizontal and vertical asymptotes for $f(x) = \frac{3x}{\sqrt{x^2 - 16}}$.
[7 marks]
- 6 The polynomial $p(x) = x^3 - 2x^2 + ax + b$, where a and b are constants, has a factor of $(x - 2)$ and leaves a remainder of a^3 when it is divided by $(x - a)$.
- (a) Find the values of a and b .
[6 marks]
- (b) Factorize $p(x)$ completely by using the values of a and b obtained from part 6(a). Hence, find the real roots of $p(x) = 0$, where a and b are not equal to zero.
[6 marks]
- 7 Given that $x = \frac{1}{\sqrt{1+t^2}}$ and $y = \frac{\sqrt{1+t^2}}{t}$, where t is a non zero parameter.
- (a) Show that $\frac{dy}{dx} = \frac{1+t^2}{t^3}$.
[6 marks]
- (b) Find $\frac{d^2y}{dx^2}$ when $t = 1$.
[6 marks]

- 8 (a) If $y = \sin(x^2 + 1)$, show that

$$x \frac{d^2 y}{dx^2} - \frac{dy}{dx} + 4x^3 y = 0.$$

[5 marks]

- (b) Find the gradient of a curve $xe^{xy} = e^{2x} - e^{3y}$ at $(0, 0)$.

[6 marks]

- 9 (a) Given $f(x) = \begin{cases} \frac{x^3 - 64}{x - 4}, & x \neq 4 \\ 40, & x = 4. \end{cases}$

- (i) Find $\lim_{x \rightarrow 4} f(x)$.

[4 marks]

- (ii) Is f continuous at $x = 4$? Give your reason.

[3 marks]

- (b) Determine the values of A and B such that the function

$$h(x) = \begin{cases} Ax - B, & x \leq -1 \\ 2x^2 + 3Ax + B, & -1 < x \leq 1 \\ 4, & x > 1. \end{cases}$$

is continuous for all values of x .

[6 marks]

10 (a) Given $\tan \frac{\pi}{3} = \sqrt{3}$ and $\tan \frac{\pi}{4} = 1$.

Express $\tan \frac{7\pi}{12}$ in the form of $a + \sqrt{b}$ where a and b are integers.

Hence, show that $\tan \left(\frac{7\pi}{6} \right) = \frac{1}{\sqrt{b}}$.

[6 marks]

- (b) Find R and α such that the expression $9\sin \theta + 12\cos \theta$ can be expressed in the form of $R\sin(\theta + \alpha)$, where $R > 0$, $0^\circ < \alpha < 90^\circ$.

Hence, if $9\sin \theta + 12\cos \theta = 5$, solve for θ in the interval $0^\circ \leq \theta \leq 360^\circ$.

[9 marks]

END OF QUESTION PAPER