QS015/1
Mathematics
Paper 1
Semester I
Session 2011/2012
2 hours

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Matematik
Kertas 1
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BAHAGIAN MATRIKULASI
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PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 1
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **15** halaman bercetak.
This question paper consists of **15** *printed pages.*

SHAMMAL

1 Solve the equation $3^{2x+1} - 28(3^x) + 9 = 0$.

[6 *marks*]

2 The functions f and g are defined as:

$$f(x) = \sqrt{x-1}, \quad x \geq 1$$
$$g(x) = x^2, \quad x \geq 0.$$

Find the inverse function, $f^{-1}(x)$ and determine its range. Then, evaluate $(f \circ g)(-2)$.

[6 *marks*]

3 The ninth term and the sum of the first fifteen terms of an arithmetic progression are 24 and 330 respectively. Find the first term, a and the common difference, d. Hence, find the least possible value n, such that the sum of the first n terms is greater than 500.

[6 *marks*]

4 Matrix A is given as $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & -3 \\ 2 & 2 & -1 \end{bmatrix}$.

(a) Given the cofactor matrix of A is $\begin{bmatrix} 3 & x+y & -2 \\ 0 & 1 & 2 \\ -3 & x^2 & -1 \end{bmatrix}$ where $x > 0$.

Determine the values of x and y.

[3 *marks*]

(b) Given $A^2 - 4A + I = 0$, show that $A^3 = 15A - 4I$ where I is the 3×3 identity matrix. Hence, find A^3.

[4 *marks*]

5 Given two complex numbers $z_1 = 5+3i$ and $z_2 = 2-i$.

(a) State $\overline{z_1}$ and $\overline{z_2}$.

[1 *mark*]

(b) Determine the value of k if $\frac{1}{z_1} = k\overline{z_1}$.

[3 *marks*]

(c) Find $z_1 z_2$. Hence, show that $\overline{z_1}\ \overline{z_2} = \overline{z_1 z_2}$.

[6 *marks*]

6 (a) Given $f(x) = e^{-x}$ and $g(x) = x^2$.

(i) Find the domain and range of f and g.

[2 *marks*]

(ii) Show that $(g \circ f)(x) = e^{-2x}$.

[2 *marks*]

(b) Given

$$h(x) = \begin{cases} e^{-2x}, & -\infty < x \le 0 \\ x+1, & x \ge 0. \end{cases}$$

(i) Find $h^{-1}(x)$.

[5 *marks*]

(ii) Sketch the graph for $h(x)$ and $h^{-1}(x)$.

[4 *marks*]

7 (a) Solve the equation $\log(x-4)+2\log 3=1+\log\left(\frac{x}{2}\right)$.

[5 *marks*]

(b) Find the solution set of the inequality

$$\left|\frac{x-3}{x+1}\right|<2.$$

[7 *marks*]

8 (a) Given that the sum of the first n terms, S_n of a series as $S_n=1-\left(\frac{1}{3}\right)^n$.

Find an expression for the *n*th term. Show that the series is a geometric series and find the sum to infinity, S_∞.

[6 *marks*]

(b) Expand $\left(1+\frac{2}{x}\right)^{\frac{1}{2}}$ in the ascending powers of x up to the term in x^3.

Hence, by substituting $x=3$, evaluate $\sqrt{\frac{5}{3}}$ correct to three decimal places.

[6 *marks*]

9 (a) A function $f(x)$ is defined by $f(x)=\dfrac{3x}{x-6}$ for $x \neq 6$.

Show that $f(x)$ is a one-to-one function.

Find the values of x such that $(f \circ f)(x)=0$.

[7 *marks*]

(b) Given $f(x)=\sqrt{1-3x}$ and $g(x)=\dfrac{x}{2}-1$.

Find $f\left(g^{-1}\left(-\dfrac{7}{2}\right)\right)$.

[6 *marks*]

10 The following table shows the quantities (unit) and the amount paid (RM) for pens bought from three shops.

Shop \ Pen	Pilot (unit)	Kilometrico (unit)	Papermate (unit)	Amount paid (RM)
S	1	p	$2p$	18.00
T	1	q	$3q$	31.00
U	1	r	$4r$	37.00

Given the price in RM per unit of pilot, kilometrico and papermate pens be x, y and z respectively.

(a) Obtain a system of linear equations to represent the given information.

[1 *mark*]

(b) Write the system in the form of a matrix equation $AX = B$ where

$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

[1 *mark*]

(c) Given the minor a_{11}, a_{21} and a_{22} of matrix A is 9, 12 and 8 respectively. Find the values of p, q and r.

[4 *marks*]

(d) Find the determinant, cofactor, adjoint and A^{-1} of matrix A. Hence, find the values of x, y and z.

[9 *marks*]

END OF QUESTION PAPER