QS025/2
Mathematics
Paper 2
Semester II
Session 2012/2013
2 hours

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Matematik
Kertas 2
Semester II
Sesi 2012/2013
2 jam

BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 2
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **19** halaman bercetak.
This question paper consists of **19** *printed pages.*

SHAMMAL

INSTRUCTIONS TO CANDIDATE:

This question paper consists of **10** questions.

Answer **all** questions.

All answers must be written in the answer booklet provided. Use a new page for each question.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Statistics

For ungrouped data, the kth percentile,

$$P_k = \begin{cases} \dfrac{x_{(s)} + x_{(s+1)}}{2}, & \text{if } s \text{ is an integer} \\ x_{([s])}, & \text{if } s \text{ is a non-integer} \end{cases}$$

where $s = \dfrac{n \times k}{100}$ and $[\ s\ ]$ = the least integer greater than k.

For grouped data, the kth percentiles, $P_k = L_k + \left[\dfrac{\left(\dfrac{k}{100} \right) n - F_{k-1}}{f_k} \right] c.$

Variance

$$s^2 = \frac{\sum f_i x_i^2 - \frac{1}{n}\left(\sum f_i x_i\right)^2}{n-1}$$

Binomial Distribution

$X \sim B(n, p)$

$P(X = x) = {}^nC_x p^x (1-p)^{n-x}, \quad x = 0, 1, 2, 3, \ldots, n$

Poisson Distribution

$X \sim P(\lambda)$

$P(X = x) = \dfrac{e^{-\lambda} \lambda^x}{x!}, \quad x = 0, 1, 2, 3, \ldots$

1 The mean and median of the ordered sample data 1, 2, 4, 7, x, y, 11, 12, 15, $2y$ are 8.7 and 8.5 respectively. Determine the values of x and y. Hence, find the variance.

[6 *marks*]

2 A box consists of five grape-flavoured sweets and four strawberry-flavoured sweets. All the sweets are of the same size. A child chooses at random four sweets from the box. Find the probability that

(a) all sweets are of the same flavour.

[3 *marks*]

(b) less than three sweets are strawberry-flavoured.

[4 *marks*]

3 A fair die is thrown once. A random variable represents the score on the uppermost face of a die. If the score is two or more, then the random variable X is the score. If the score is one, the die is to be thrown once again and the random variable X is the sum of scores of the two throws. Construct the probability distribution table for X.

[6 *marks*]

4 The number of motorcycles arriving at the main entrance of a university during peak hours has a Poisson distribution with mean three per minute. Find the probability that

(a) at most one motorcycle will arrive in one minute.

[3 *marks*]

(b) exactly five motorcycles will arrive in two minutes.

[3 *marks*]

5 The following table gives the cumulative frequency distribution for the weights (kg) of fifty hampers during a festival at a supermarket.

Weight (kg)	Cumulative frequency
< 2.5	0
< 5.5	5
< 8.5	15
< 11.5	28
< 14.5	40
< 17.5	50

(a) Find the mean, median and standard deviation.

[7 *marks*]

(b) Hence, calculate the Pearson's coefficient of skewness and interpret your answer.

[3 *marks*]

(c) State with reason whether mean or median is a better measure of location.

[1 *mark*]

6 A security code is to be formed by using three alphabets and four digits chosen from the alphabets {a, b, c, d, e} and digits {1, 2, 3, 4, 5, 6}. All the digits and alphabets can only be used once. Find the number of different ways the security code can be formed if

(a) there is no restriction imposed.

[3 *marks*]

(b) all alphabets are next to each other and all digits are next to each other.

[3 *marks*]

(c) it consists of at least two consonants.

[5 *marks*]

7 Every year two teams, Unggul and Bestari meet each other in a debate competition. Past results show that in years when Unggul win, the probability of them winning the next year is 0.6 and in years when Bestari win, the probability of them winning the next year is 0.5. It is not possible for the competition to result in a tie. Unggul won the competition in 2011.

(a) Construct a probability tree diagram for the three years up to 2014.

[2 *marks*]

(b) Find the probability that Bestari will win in 2014.

[3 *marks*]

(c) If Bestari wins in 2014, find the probability that it will be their first win for at least three years.

[3 *marks*]

(d) Assuming that Bestari wins in 2014, find the smallest value of n such that the probability of Unggul wins the debate competition for n consecutive years after 2014 is less than 0.05.

[5 *marks*]

8 A discrete random variable X has a probability distribution function

$$p(x) = \begin{cases} \dfrac{2^{5-x}}{32}, & x = 1,\ 2,\ 3,\ 4 \\ k, & x = 5 \end{cases}$$

where k is a constant.

(a) Show that $k = \dfrac{1}{16}$.

[2 *marks*]

(b) Find $P(1 \leq X < 3)$.

[2 *marks*]

(c) Calculate the mean of X and hence, calculate $E(2X - 3)$.

[4 *marks*]

(d) Find the variance of X and hence, calculate $\text{Var}(9 - 2X)$.

[5 *marks*]

9 The continuous random variable X has the probability density function

$$f(x)=\begin{cases}\frac{6}{5}x, & 0\le x<1,\\ \frac{6}{5}(2-x)^2, & 1\le x<2,\\ 0, & \text{otherwise.}\end{cases}$$

(a) Find the cumulative distribution function of X.

[5 *marks*]

(b) Find

(i) $P(0.5<X<1.5)$.

[2 *marks*]

(ii) $P(X>1.5)$.

[2 *marks*]

(c) Calculate the median of X correct to three decimal places.

[3 *marks*]

10 The registration record of a private college indicates that 40% of its new intakes are international students and the remaining are local students.

(a) If 20 new students are randomly selected and the number of local students are noted, find the probability that there are

(i) equal number of local and international students.

[2 *marks*]

(ii) not less than 9 local students.

[4 *marks*]

(b) Exactly 100 new students are randomly selected. By using a suitable approximate distribution,

(i) find the probability that between 38 and 46 are international students.

[5 *marks*]

(ii) determine the value m such that the probability that the number of international students is at most m is 0.993.

[4 *marks*]

END OF QUESTION PAPER