QS025/1	**QS025/1**
Mathematics	**Matematik**
Paper 1	**Kertas 1**
Semester II	**Semester II**
Session 2012/2013	**Sesi 2012/2013**
2 hours	**2 jam**

BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 1
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **15** halaman bercetak.
This question paper consists of **15** *printed pages.*

SHAMMAL

INSTRUCTIONS TO CANDIDATE:

This question paper consists of **10** questions.

Answer **all** questions.

All answers must be written in the answer booklet provided. Use a new page for each question.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Trigonometry

$$\sin\,(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos\,(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan\,(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\sin^2 A = \frac{1 - \cos 2A}{2}$$

$$\cos^2 A = \frac{1 + \cos 2A}{2}$$

LIST OF MATHEMATICAL FORMULAE

Differentiation and Integration

$$\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\text{cosec} x) = -\text{cosec } x \cot x$$

$$\int f'(x) e^{f(x)}\, dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\, \text{dx} = \ln |f(x)| + c$$

$$\int u\, dv = uv - \int v\, du$$

Sphere	$V = \frac{4}{3}\pi r^3$	$S = 4\pi r^2$
Right circular cone	$V = \frac{1}{3}\pi r^2 h$	$S = \pi r^2 + \pi r h$
Right circular cylinder	$V = \pi r^2 h$	$S = 2\pi r^2 + 2\pi r h$

LIST OF MATHEMATICAL FORMULAE

Numerical Methods

Iteration Method:

$$x_{n+1} = g(x_n), \quad n = 1, 2, 3, \ldots \text{ where } \left| g'(x_1) \right| < 1$$

Newton-Raphson Method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 1, 2, 3, \ldots.$$

Conics

Circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$$

$$r = \sqrt{f^2 + g^2 - c}$$

$$d = \sqrt{a^2 + b^2 + 2ga + 2fb + c}$$

Parabola:

$$(x-h)^2 = 4p(y-k)$$

$$(y-k)^2 = 4p(x-h)$$

$$F(h+p, k) \text{ or } F(h, k+p)$$

Ellipse:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$F(h \pm c, k) \text{ or } F(h, k \pm c)$$

1 Find the values of A and B if $\dfrac{2x^2+4x+3}{x^2+x+1}=\dfrac{A(x^2+x+1)+B(2x+1)}{x^2+x+1}$.

Hence, find $\displaystyle\int \frac{2x^2+4x+3}{x^2+x+1}\,dx$.

[7 *marks*]

2 Solve the differential equation $x\dfrac{dy}{dx}+y=x\sin x,\ y(\pi)=1$.

[7 *marks*]

3 Find the center and radius of the circle $x^2+y^2+2x=4$. Obtain the equation of the tangent to the circle at the point $(0,2)$.

[7 *marks*]

4 (a) If $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are nonzero vectors, show that $\underset{\sim}{u}\bullet(\underset{\sim}{u}\times\underset{\sim}{v})=0$.

[3 *marks*]

(b) Find a unit vector perpendicular to $\underset{\sim}{u}=-2\underset{\sim}{i}+3\underset{\sim}{j}-3\underset{\sim}{k}$ and $\underset{\sim}{v}=2\underset{\sim}{i}-\underset{\sim}{k}$.

[4 *marks*]

5 (a) Show that $x^4=3x^2-1$ has a solution on the interval $(1,2)$.

[2 *marks*]

(b) Use Newton-Raphson's method with $x_0=1$ to estimate the solution for part (a) correct to four decimal places.

[7 *marks*]

6 (a) Solve the differential equation $\frac{dy}{dx}=\frac{4x^3}{3y^2}$, given that $y=2$ when $x=0$.

[4 *marks*]

(b) Assume that *P(t)* represents the size of a population at any time *t* and the increment in population size at time *t* can be modeled by the differential equation $\frac{dP(t)}{dt}=0.005P(t)$ with an initial condition $P(0)=1500$. Determine the size of this population after 10 years.

[6 *marks*]

7 (a) Find $\int \sin^3 x \cos^4 x \, dx$ by using the substitution $u=\cos x$.

[6 *marks*]

(b) Evaluate $\int_1^e x \ln x \, dx$.

[6 *marks*]

8 An equation $x^2-4x-4y+8=0$ represents a parabola.

(a) Determine the vertex, focus and directrix of the parabola.

[6 *marks*]

(b) Show that the tangent lines to the parabola at the points $A(-2,5)$ and $B(3,\frac{5}{4})$ intersect at the right angle.

[7 *marks*]

9 (a) Sketch and shade the region R bounded by the curves $y = x^2 + 2$, the line $2y - x = 2$, $x = 0$ and $x = 2$. Hence, find the area of R.

[7 *marks*]

(b) If the region R in part (a) is rotated through 2π radian about the $x-$axis, find the volume of the solid generated.

[6 *marks*]

10 The plane Π_1 contains a line L with vector equation $\underset{\sim}{r} = t\underset{\sim}{j}$ and a point $P(3, -1, 2)$.

(a) Find a Cartesian equation of Π_1.

[6 *marks*]

(b) Given a second plane Π_2 with equation $x + 2y + 3z = 4$, calculate the angle between Π_1 and Π_2.

[4 *marks*]

(c) Find a vector equation for the line of intersection of Π_1 and Π_2.

[5 *marks*]

END OF QUESTION PAPER