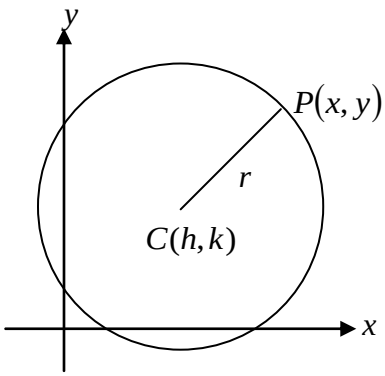
LECTURE 1 OF 5
TOPIC 4 : CONICS
SUBTOPIC : 4.1 Circles
LEARNING OUTCOMES : At the end of the lesson, students should be able to
a) determine the equation of a circle.

a) Determine the equation of a circle

A circle is a set of points in a plane that has a fixed distance from a given fixed point. The fixed point is called the **centre** and the fixed distance is called the **radius**.



Consider a circle having a radius of length r and a centre at (h,k) on a coordinate system, as shown in figure above. For any point P on the circle with coordinate (x,y), the length of a radius, denoted by r, can be expressed as $r=\sqrt{(x-h)^2+(y-k)^2}$. Thus, squaring both sides of the equation we obtain the **standard form** of the equation of a circle.

$$\boxed{(x-h)^2+(y-k)^2=r^2} \qquad (1)$$

By expanding standard equation of circle $(x-h)^2+(y-k)^2=r^2$,
we get $x^2+y^2-2hx-2ky+h^2+k^2-r^2=0$.
Now substituting $g=-h$, $f=-k$ and $c=h^2+k^2-r^2$, so we get

$$\boxed{x^2+y^2+2gx+2fy+c=0}$$

This is a **general form** of circle.

Completing the square, we get

$$(x+g)^2+(y+f)^2=g^2+f^2-c. \qquad (2)$$

Comparing (1) and (2), we can determine that its centre is at $(-g,-f)$ and the length of the radius is $r=\sqrt{g^2+f^2-c}$.

Example 1
Find the standard equation of the circle having its centre at $(2,3)$ and radius of length 3 units.

Example 2
The centre of a circle is at (0, 0) and passing through (4, 3). Find the standard equation of circle.

Example 3
Find the general equation of a circle having its centre at $(-3,5)$ and radius of length 4 units.

Example 4
Find the general equation of the circle passing through the points (0,1), (4,3) and (1,-1).

Example 5
Find the general equation of the circle passing through the points $(1,1)$ and $(3,2)$ with diameter on $y-3x+7=0$

Example 6
Find the general equation of the circle having AB as diameter where A is the point $(1,8)$ and B is the point $(3,14)$.

LECTURE 2 OF 5
TOPIC 4 : CONICS
SUBTOPIC : 4.1 Circles
LEARNING OUTCOMES : At the end of the lesson, students should be able to

b) determine the centre and radius of a circle.
c) find the points of intersection of two circles, and a circle and a line.

b) Determine the centre and radius of a circle

Example 1
Find the centre and the radius of the circle $x^2 + y^2 - 6x + 4y + 9 = 0$.

Example 2
Find the centre and radius of the circle $x^2 + y^2 - 6x + 12y - 2 = 0$.

c) Find the points of intersection of two circles, and a circle and a line

The type of intersection between two circles, and a circle and a line can be determine by using discriminant $b^2 - 4ac$.
If $b^2 - 4ac > 0$, then the two circles intersect at two points.
If $b^2 - 4ac = 0$, then the two circles intersect at one point.
If $b^2 - 4ac < 0$, then the two circles do not intersect each other.

Example 3
Find the points of intersection of circles $x^2 + y^2 - 4x + 6y - 12 = 0$ and $x^2 + y^2 - 5x + 3y - 4 = 0$

Example 4
Determine the points of intersection of the circle $x^2 + y^2 = 10$ with the line $y = 5 - 2x$.

Shortest distance from a point to a line

The **shortest distance** or **perpendicular distance** from a point (h, k) to a straight line $ax + by + c = 0$ is given by $d = \dfrac{|ah + bk + c|}{\sqrt{a^2 + b^2}}$.

Example 5
The equation of circle P is given by $x^2 + y^2 - 4x + 6y - 12 = 0$. Find the perpendicular distance from the centre P to the line $3x + 4y = 19$.

LECTURE 3 OF 5
TOPIC 4 : CONICS
SUBTOPIC : 4.1Circles
LEARNING OUTCOMES : At the end of the lesson, students should be able to

d) find the equation of tangents and normals to a circle.
e) find the length of a tangent from a point to a circle.

d) Find the equation of tangents and normal to a circle

i) **The point** $P(x_1, y_1)$ **lie on circle**

The equation of the tangent at any point $P(x_1, y_1)$ on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$.

Example 1
Find the equation of the tangent and normal to the circle $x^2 + y^2 - 6x - 10y - 82 = 0$ at $S(-1,-5)$.

ii) **The point** $P(x_1, y_1)$ **not lie on circle**

Example 2
Find the equation of the tangent to a circle $x^2 + y^2 = 1$ from point (0, 3).

e) Find the length of a tangent from a point to a circle

The length of a tangent from a point to a circle can be found by using a diagram and Pythagoras theorem.

Example 3
Find the length of the tangent from the point P(4, 6) to the circle $x^2 + y^2 - 4x - 2y = 6$

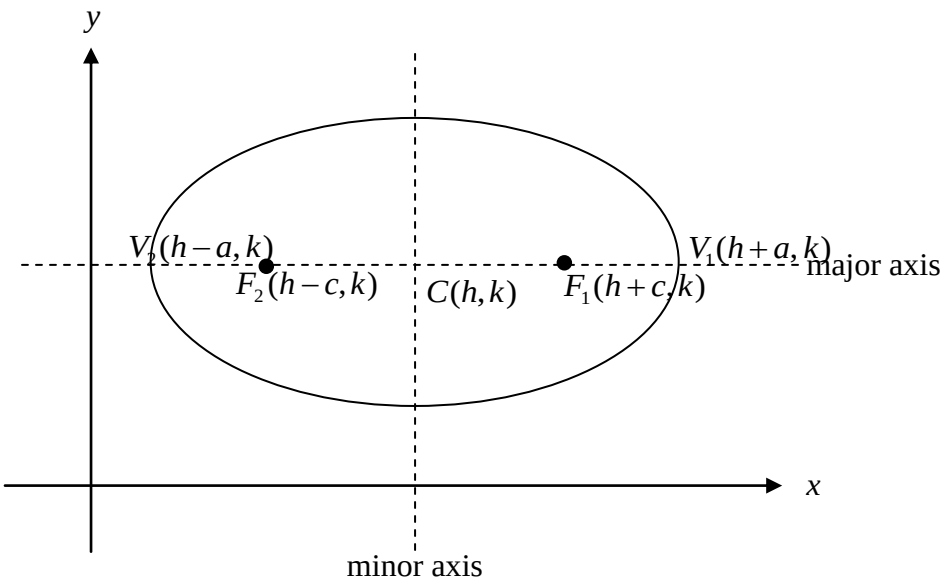
LECTURE 4 OF 5
TOPIC 4 : CONICS
SUBTOPIC : 4.2 Ellipses
LEARNING OUTCOMES : At the end of the lesson, students should be able to

a) determine the equation of the ellipse with centre (h,k) and foci $(h \pm c, k)$ or $(h, k \pm c)$.
b) determine all the vertices, foci, major and minor axes.
c) determine the centre and foci of an ellipse by completing the square.

The equation of the ellipse with centre (h,k) and foci $(h \pm c, k)$

$$\boxed{\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1}$$

where $c^2 = a^2 - b^2$ if $a > b > 0$



Centre	(h,k)
Vertices	$V_1(h+a,k)$ and $V_2(h-a,k)$
Foci	$F_1(h+c,k)$ and $F_2(h-c,k)$
Length of the major axis	$2a$
Length of the minor axis	$2b$

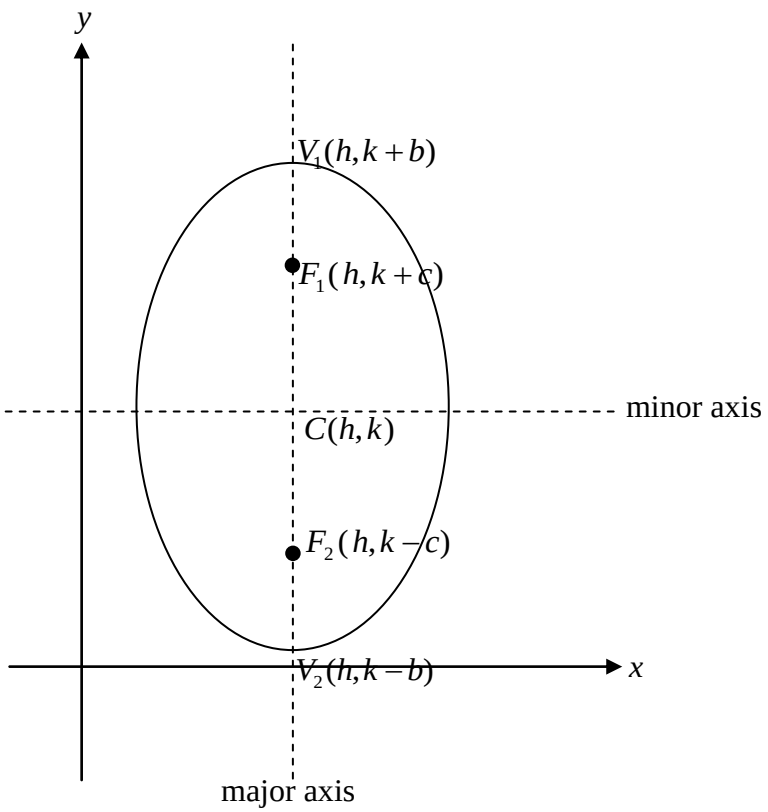
Example 1
Find the centre, vertices, foci, length of major axis and minor axis of the ellipse $\frac{(x-1)^2}{25}+\frac{y^2}{16}=1$. Hence, sketch the graph.

Example 2
Write the equation of the ellipse that has vertices at $(-3,-5)$ and $(7,-5)$ and foci at $(-1,-5)$ and $(5,-5)$.

The equation of the ellipse with centre (h,k) **and foci** $(h,\ k\pm c)$

$$\boxed{\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1}$$

where $c^2=b^2-a^2$ if $b>a>0$



Centre	(h,k)
Vertices	$V_1(h,k+b)$ and $V_2(h,k-b)$
Foci	$F_1(h,k+c)$ and $F_2(h,k-c)$
Length of the major axis	$2b$
Length of the minor axis	$2a$

Example 3
Find the equation of ellipse which centre at $(3,1)$ with the major axis parallel to the *y*-axis, the length of the major axis is 10 units and the length of minor axis is 6 units. Sketch the ellipse.

Example 4
Find the equation of ellipse with vertex $(8,5)$, length of minor axis is 4 and centre at $(8,1)$

Determine the centre and foci of an ellipse by completing the square

Example 5
Sketch the graph of the equation $16x^2 + 9y^2 + 64x - 18y - 71 = 0$.

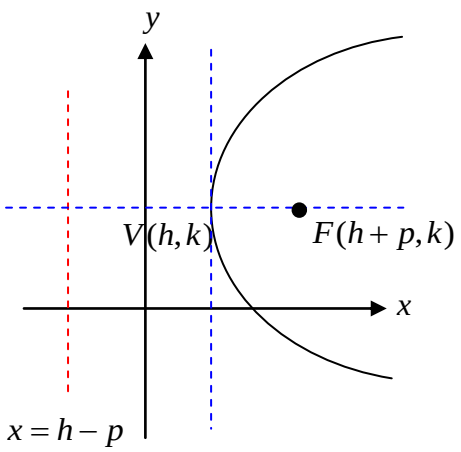
LECTURE 5 OF 5
TOPIC 4 : CONICS
SUBTOPIC : 4.3 Parabola
LEARNING OUTCOMES : At the end of the lesson, students should be able to

a) find the equation of the parabola with vertex (h,k) and focus $(h+p,k)$ or $(h,k+p)$.
b) determine the vertex, focus and directrix of a parabola by completing the square.

The equation of the parabola with vertex (h,k) and focus $(h+p,k)$

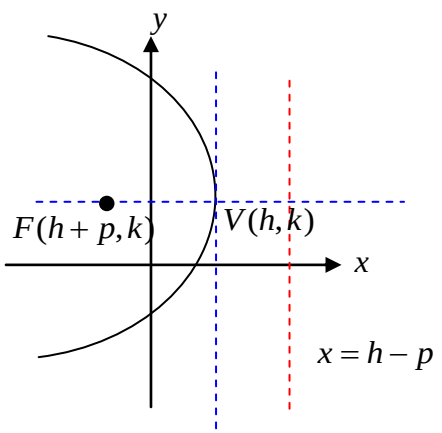
$$\boxed{(y-k)^2 = 4p(x-h)}$$

i) when $p>0$ (opens right)



Vertex	$V(h,k)$
Focus	$F(h+p,k)$
Directrix	$x=h-p$

ii) when $p<0$ (opens left)

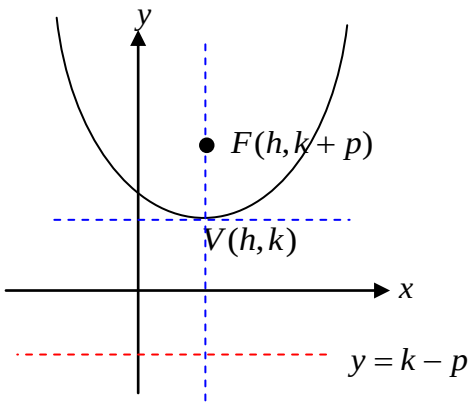


Vertex	$V(h,k)$
Focus	$F(h+p,k)$
Directrix	$x=h-p$

The equation of the parabola with vertex (h,k) **and focus** $(h,k+p)$

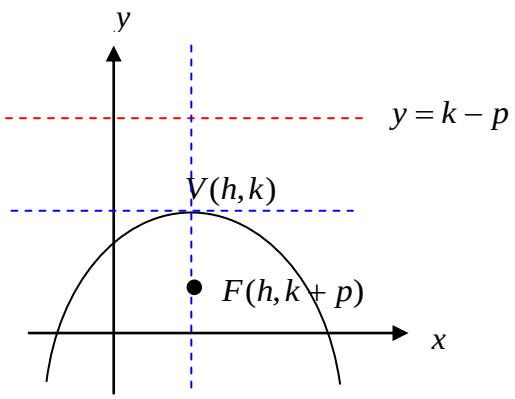
$$\boxed{(x-h)^2 = 4p(y-k)}$$

i) when $p > 0$ (opens upward)



Vertex	$V(h,k)$
Focus	$F(h,k+p)$
Directrix	$y = k - p$

ii) when $p < 0$ (opens downward)



Vertex	$V(h,k)$
Focus	$F(h,k+p)$
Directrix	$y = k - p$

The equation $(y-k)^2 = 4p(x-h)$ or $(x-h)^2 = 4p(y-k)$ is known as **standard form** of parabola equation and it can be written in the **general form** as
$Ay^2 + Dx + Ey + F = 0$ or $Ax^2 + Dx + Ey + F = 0$

Example 1
State the vertex, focus and directrix for each of the following
a) $(y-2)^2 = 12(x-3)$
b) $(x-1)^2 = 5(y+2)$

Example 2
Sketch the graph of $(y-2)^2 = 12(x-1)$ showing clearly the vertex, focus and directrix of the curve.

Example 3
Find the equation of a parabola which satisfies the following conditions, vertex $(-1,-2)$, its axis parallel to the y-axis and the parabola passes through the point (3,6).

Determine the vertex, focus and directrix of a parabola by completing the square

Example 4
Express the equation of given parabola below in standard form. For each parabola state the coordinates of the vertex, focus and the equation of the directrix. Hence, sketch each graph.

a) $x^2 - 8x + 4y + 12 = 0$

b) $y^2 + 8y - 2x + 22 = 0$