1. a) Solve the equation $2^{2(x+1)} + 3(2^x) = 1$ **[4 marks]**

b) Given $(4-i)x - 3y = 5 + i$. Find the values of x and y where $x, y \in R$ **[3 marks]**

2. a) Simplify $\frac{1+\sqrt{2}}{1-\sqrt{2}} + \frac{1-\sqrt{2}}{1+\sqrt{2}}$ **[3 marks]**

b) Solve $\ln x - \frac{3}{\ln x} = -2$ **[4 marks]**

3. Solve the following

a) $2x^2 - 3 < 3x + 2$ **[5 marks]**

b) $|2x+3| < 13$ **[4 marks]**

4. A geometric progression is given by $2, 2(0.9), 2(0.9)^2,...$

a) Show that $S_n = 20(1-0.9^n)$. Hence, find $\frac{S_5}{S_3}$. **[5 marks]**

b) If S_∞ is the sum to infinity of the series, find the smallest value of n such that

$S_\infty - S_n < 4$. **[4 marks]**

5. Express $(8-3x)^{-\frac{1}{3}}$ in the form of $a(1-bx)^{-\frac{1}{3}}$ and hence expand $(8-3x)^{-\frac{1}{3}}$ as a series of ascending powers of x, up to the term in x^2.

a) State the range of x for which this expansion is valid. **[5 marks]**

b) By using $x = 1$, find the approximation for $\sqrt[3]{5}$ correct to one decimal place. **[4 marks]**

6. Find $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 2 & 3 & 0 \end{bmatrix}\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$.

Hence solve the system,

$x - y + z = 1$

$2y - z = 4$

$2x + 3y = 7$

[9 marks]

Final Answers

1. a) -2 b) $x = -1, y = -3$

2. a) -6 b) e^{-3} , e

3. a) $\left(-1, \frac{5}{2}\right)$ b) $(-8,5)$

4. a) 1.511 b) n=16

5. $\frac{1}{2}\left(1 - \frac{3}{8}x\right)^{-\frac{1}{3}}$; $\frac{1}{2} + \frac{1}{16}x + \frac{1}{64}x^2$

a) $\left(-\frac{8}{3}, \frac{8}{3}\right)$

b) 1.7

6. $x = 8, y = -3, z = -10$

Answer Scheme

1. a) $2^{2(x+1)}+3(2^x)=1$

$2^{2x+2}+3(2^x)=1$

$(2^x)^2 2^2+3(2^x)=1$

Let $2^x=a$

$4a^2+3a=1$

$4a^2+3a-1=0$

$(4a-1)(a+1)=0$

$a=\frac{1}{4}, a=-1$

$2^x=2^{-2}$ $\qquad$ $2^x=-1$

$x=-2$ $\qquad$ no solution

b) $(4-i)x-3y=5+i$

$4x-xi-3y=5+i$

$(4x-3y)-xi=5+i$

$x=-1,$ $\qquad$ $4(-1)-3y=5$

$-3y=5+4$

$y=-3$

2. a) $\frac{1+\sqrt{2}}{1-\sqrt{2}}+\frac{1-\sqrt{2}}{1+\sqrt{2}}$

$\frac{(1+\sqrt{2})(1+\sqrt{2})+(1-\sqrt{2})(1-\sqrt{2})}{(1-\sqrt{2})(1+\sqrt{2})}$

$\frac{1+2\sqrt{2}+2+1-2\sqrt{2}+2}{1-2}$

-6

b) $\ln x - \dfrac{3}{\ln x} = -2$

Let $\ln x = a$

$a - \dfrac{3}{a} = -2$

$a^2 - 3 = -2a$

$a^2 + 2a - 3 = 0$

$(a+3)(a-1) = 0$

$a = -3, a = 1$

$\ln x = -3, \qquad \ln x = 1$

$x = e^{-3} \qquad x = e^{1}$

3. a) $2x^2 - 3 < 3x + 2$

$2x^2 - 3 - 3x - 2 < 0$

$2x^2 - 3x - 5 < 0$

$(2x-5)(x+1) < 0$

C.V ; $x = \dfrac{5}{2}, -1$

By using graphical : $\left(-1, \dfrac{5}{2}\right)$

b) $|2x+3| < 13$

$2x + 3 < 13$ and $2x + 3 > -13$

$2x < 10$ $\qquad 2x > -16$

$x < 5$ $\qquad x > -8$

By using number line: $-8 < x < 5$

4. a) $2,\ 2(0.9),\ 2(0.9)^2, ...$

$a = 2, \quad r = 0.9$

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_n = \dfrac{2(1-(0.9)^n)}{1-0.9}$

$$S_n = \frac{2(1-(0.9)^n)}{0.1}$$

$$\mathrm{S_n} = 20(1-0.9^{\mathrm{n}}) \text{ Shown}$$

b) $$\frac{\mathrm{S_5}}{\mathrm{S_3}} = \frac{20\left(1-0.9^5\right)}{20(1-0.9^3)} = 1.511$$

$$\mathrm{S_\infty} - \mathrm{S_n} < 4$$

$$\frac{2}{1-0.9} - \frac{2(1-0.9^n)}{1-0.9} < 4$$

$$\frac{2}{0.1} - \frac{2(1-0.9^n)}{0.1} < 4$$

$$20 - 20(1-0.9^n) < 4$$

$$20(1-(1-0.9^n)) < 4$$

$$20(0.9^n) < 4$$

$$(0.9^n) < \frac{1}{5}$$

$$\log(0.9^n) < \log\frac{1}{5}$$

$$n\log(0.9) < \log\frac{1}{5}$$

$$n > \frac{\log\frac{1}{5}}{\log(0.9)} = 15.3$$

$$n = 16$$

5.

a) $(8-3x)^{-\frac{1}{3}} = [8(1-\frac{3}{8}x)]^{-\frac{1}{3}}$

$8^{-\frac{1}{3}}(1-\frac{3}{8}x)^{-\frac{1}{3}} = \frac{1}{2}(1-\frac{3}{8}x)^{-\frac{1}{3}}$

$$=\frac{1}{2}\left[1+\left(-\frac{1}{3}\right)\left(-\frac{3}{8}x\right)+\frac{\left(-\frac{1}{3}\right)\left(-\frac{1}{3}-1\right)\left(-\frac{3}{8}x\right)^2}{2!}+\ldots\right]$$

$$=\frac{1}{2}\left[1+\frac{1}{8}x+\frac{1}{32}x^2+\ldots\right] = \frac{1}{2}+\frac{1}{16}x+\frac{1}{64}x^2+\ldots$$

Expansion is valid for $-\frac{8}{3} < x < \frac{8}{3}$ or $|x| < \frac{8}{3}$

(b) By substituting $x = 1$,

$[8-3(1)]^{-\frac{1}{3}} = \frac{1}{2}+\frac{1}{16}(1)+\frac{1}{64}(1)^2+\ldots$

$(5)^{-\frac{1}{3}} = 0.578$

$\sqrt[3]{5} = 1.7$ (1 decimal place)

6. $$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 2 & 3 & 0 \end{bmatrix}\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$AX = B$

$X = A^{-1}B$ $\therefore A^{-1} = B$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 2 & 3 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -3 \\ -10 \end{bmatrix}$$