1. a) Simplify $\dfrac{1+\sqrt{2}}{1-\sqrt{2}}+\dfrac{1-\sqrt{2}}{1+\sqrt{2}}$. **[2 marks]**

   b) Solve $\log_2 x + 3\log_x 2 = 4$. **[4 marks]**

2. Find the solution for $\dfrac{3x+4}{x-7} \geq 2$. **[6 marks]**

3. a) By mean of substitution $u = x^{-\frac{1}{2}}$, or otherwise, solve the equation $x^{-\frac{1}{2}} + 2x^{-1} = 15$. **[5 marks]**

   b) A complex number $z = x + iy$ and its conjugate $\bar{z}$ are related by $\left|z-1\right| = \left|\bar{z}+2i\right|$. Given the argument of z is $\tan^{-1}(1.5)$. Find the complex number z.

   [6 marks]

4. The sum of the first two terms and the sum to infinity of a geometric progression are $\dfrac{48}{7}$ and 7 respectively. Find the values of the common ratio r, and the first term when r is positive.

   [5 marks]

5. a) The term independent of *x* in the expansion of $\left(x+\dfrac{1}{ax^2}\right)^9$ is $\dfrac{21}{2}$. Find *a*.

   [4 marks]

   b) In geometric progression, the first term is 18 and the fourth term is $-\dfrac{2}{3}$. Find the sum of the first *n* terms, S_n. Find the sum to infinity of the progression and the least value of *n* such that of the difference between S_n and the sum to infinity is less than 0.0001.

   [6 marks]

6. Given $A = \begin{bmatrix} 3 & 1 & -2 \\ 1 & 2 & 0 \\ 4 & 3 & -4 \end{bmatrix}$

Find

a) $|A|$

b) $adj\ A$

c) By using (a) and (b), find A^{-1}

Hence, solve the following system of linear equations

$$3x + y - 2z = -1$$
$$x + 2y = 0$$
$$4x + 3y - 4z = 2$$

[12 marks]

Final Answers

1. a) -6
   b) $x = 2$ or $x = 8$

2. $x \le -18$ or $x > 7$

3. a) $\dfrac{4}{25}$
   b) $z = \dfrac{3}{4} + \dfrac{9}{8}i$

4. $r = \dfrac{1}{7}$ or $-\dfrac{1}{7}$; $a = 6$

5. a) 2
   b) $S_n = \dfrac{27}{2}\left[1 - \left(-\dfrac{1}{3}\right)^n\right]$, $S_\infty = \dfrac{27}{2}$, $n = 11$

6. a) -10 b) $adjA = \begin{bmatrix} -8 & -2 & 4 \\ 4 & -4 & -2 \\ -5 & -5 & 5 \end{bmatrix}$ c) $x = -\dfrac{8}{5},\ y = \dfrac{4}{5}, z = -\dfrac{3}{2}$

Answer Scheme

1. a) $\dfrac{1+\sqrt{2}}{1-\sqrt{2}}+\dfrac{1-\sqrt{2}}{1+\sqrt{2}} = \dfrac{\left(1+\sqrt{2}\right)\left(1+\sqrt{2}\right)+\left(1-\sqrt{2}\right)\left(1-\sqrt{2}\right)}{\left(1-\sqrt{2}\right)\left(1+\sqrt{2}\right)}$

$$= \frac{1+2\sqrt{2}+2+1-2\sqrt{2}+2}{1-2} = \frac{6}{-1} = -6\,.$$

b) $\log_2 x + 3\log_x 2 = 4$

$\log_2 x + \dfrac{3}{\log_2 x} = 4$

$(\log_2 x)^2 - 4\log_2 x + 3 = 0$

$(\log_2 x - 3)(\log_2 x - 1) = 0$

$\log_2 x = 3$ or $\log_2 x = 1$

$x = 2^3$ or $x = 2^1$

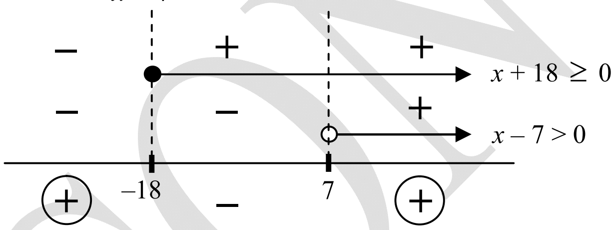
$\therefore\ x = 2,\ 8$

2. $\dfrac{3x+4}{x-7} \ge 2$

$\dfrac{3x+4}{x-7} - 2 \ge 0$

$\dfrac{3x+4-2(x-7)}{x-7} \ge 0$

$\dfrac{x+18}{x-7} \ge 0$



$\therefore\ x \le -18$ or $x > 7$

3. a) $u = x^{-\frac{1}{2}}$ so $u^2 = x^{-1}$

hence, $x^{-\frac{1}{2}} + 2x^{-1} = 15$

$u + 2u^2 = 15$

$2u^2 + u - 15 = 0$

$(2u-5)(u+3) = 0$

$u = \dfrac{5}{2}$ or $u = -3$

$x^{-\frac{1}{2}} = \dfrac{5}{2}$ or $x^{-\frac{1}{2}} = -3$

$\dfrac{1}{\sqrt{x}} = \dfrac{5}{2}$ or $\dfrac{1}{\sqrt{x}} = -3$ (*rejected*)

$$\frac{1}{x}=\frac{25}{4}$$
$$\therefore\ x=\frac{4}{25}$$

b) $z=x+iy$ then $z^*=x-iy$

$$|z-1|=|z^*+2i|$$
$$|x+iy-1|=|x-iy+2i|$$
$$\sqrt{(x-1)^2+y^2}=\sqrt{x^2+(2-y)^2}$$
$$x^2-2x+1+y^2=x^2+4-4y+y^2$$
$$4y-2x=3 \text{ --------------------- (1)}$$
$$\arg(z)=\tan^{-1}\left(\frac{y}{x}\right)=\tan^{-1}(1.5)$$
$$\frac{y}{x}=\frac{3}{2}$$
$$y=\frac{3}{2}x \text{ --------------------- (2)}$$

Solve (1) and (2) $\Rightarrow$ $x=\frac{3}{4}$ and $y=\frac{9}{8}$

$$\therefore\ z=\frac{3}{4}+\frac{9}{8}i$$

4. $S_2=\dfrac{a(1-r^2)}{1-r}=\dfrac{48}{7}$ --------------------- (1)

$S_\infty=\dfrac{a}{1-r}=7$ --------------------- (2)

By substituting (2) into (1):

$$7(1-r^2)=\frac{48}{7}$$
$$1-r^2=\frac{48}{49}$$
$$r^2=\frac{1}{49}$$
$$\therefore\ r=\pm\frac{1}{7}$$

For $r=\frac{1}{7}$ $\Rightarrow$ $\dfrac{a}{1-\frac{1}{7}}=7$

$$\therefore\ a=6$$

5. a) $\left(x+\dfrac{1}{ax^2}\right)^9$

$$T_{r+1} = {}^9C_r\ x^r\left(\frac{1}{ax^2}\right)^{9-r} = {}^9C_r\left(\frac{1}{a}\right)^{9-r} x^{r-18+2r} = {}^9C_r\left(\frac{1}{a}\right)^{9-r} x^{3r-18}$$

Term independent of x when $3r-18=0$

$r=6$

$$\Rightarrow \quad {}^9C_6\left(\frac{1}{a}\right)^3 = \frac{21}{2}$$

$$\frac{1}{a^3}=\frac{1}{8}$$

$\therefore\ a=2$

b) $a=18$, $T_4 = ar^3 = -\dfrac{2}{3}$

$$18r^3 = -\frac{2}{3}$$

$$r^3 = -\frac{1}{27} \quad\Rightarrow\quad r=-\frac{1}{3}$$

$$\therefore \quad S_n = \frac{18\left[1-\left(-\frac{1}{3}\right)^n\right]}{1-\left(-\frac{1}{3}\right)} = \frac{27}{2}\left[1-\left(-\frac{1}{3}\right)^n\right].$$

$$\Rightarrow \quad S_\infty = \frac{a}{1-r} = \frac{18}{1-\left(-\frac{1}{3}\right)} = \frac{27}{2}.$$

$$\Rightarrow \quad \left|S_\infty - S_n\right| < 0.0001$$

$$\left|\frac{27}{2}-\frac{27}{2}\left[1-\left(-\frac{1}{3}\right)^n\right]\right| < 0.0001$$

$$\left|\frac{27}{2}\left(-\frac{1}{3}\right)^n\right| < 0.0001$$

$$\frac{27}{2}\left(\frac{1}{3}\right)^n < 0.0001$$

$$\left(\frac{1}{3}\right)^n < \frac{0.0002}{27}$$

$$n\log\left(\frac{1}{3}\right) < \log\left(\frac{0.0002}{27}\right)$$

$$n > \frac{\log\left(\frac{0.0002}{27}\right)}{\log\left(\frac{1}{3}\right)}$$

$$n > 10.75$$

$$\therefore \quad n = 11.$$

6. a) $|A| = -1(-4+6) + 2(-12+8) = -10$

b)

$$\left[C_{ij}\right] = \begin{bmatrix} +(-8-0) & -(-4-0) & +(3-8) \\ -(-4+6) & +(-12+8) & -(9-4) \\ +(0+4) & -(0+2) & +(6-1) \end{bmatrix} = \begin{bmatrix} -8 & 4 & -5 \\ -2 & -4 & -5 \\ 4 & -2 & 5 \end{bmatrix}$$

$$\therefore adjA = \left[C_{ij}\right]^T = \begin{bmatrix} -8 & -2 & 4 \\ 4 & -4 & -2 \\ -5 & -5 & 5 \end{bmatrix}$$

c) $$A^{-1} = \frac{1}{|A|} adjA = \frac{1}{-10}\begin{bmatrix} -8 & -2 & 4 \\ 4 & -4 & -2 \\ -5 & -5 & 5 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} & -\frac{2}{5} \\ -\frac{2}{5} & \frac{2}{5} & \frac{1}{5} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & -2 \\ 1 & 2 & 0 \\ 4 & 3 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-10}\begin{bmatrix} -8 & -2 & 4 \\ 4 & -4 & -2 \\ -5 & -5 & 5 \end{bmatrix}\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} -\frac{8}{5} \\ \frac{4}{5} \\ -\frac{3}{2} \end{bmatrix}$$

$$x = -\frac{8}{5},\ y = \frac{4}{5}, z = -\frac{3}{2}$$