1. a) Simplify $\dfrac{4+7i}{2+5i}$ in the form a+bi **[3 marks]**

   b) Solve the equation $5^{2x} + 4(5^x) - 5 = 0$ **[4 marks]**

2. Solve $\dfrac{5}{x} < 2 + 3x$ **[5 marks]**

3. The sum of the first four terms of a geometric series with common ratio $-\dfrac{1}{2}$ is 30.

   Determine the tenth term and the infinite sum, S_∞. **[7 marks]**

4. Expand $\dfrac{1}{(3-x)^3}$ up to the term x^3 and determine the interval of x for which the expansion is valid. Hence, approximate $\dfrac{1}{(2.9)^3}$,correct to four decimal places.

   [7 marks]

5. Solve:

   a) $\log_3 x - 4\log_x 3 + 3 = 0$ **[6 marks]**

   b) $\sqrt{3x+1} - \sqrt{2x-1} = \sqrt{x+2}$ **[6 marks]**

.

6. Given $A = \begin{bmatrix} -1 & 0 & -2 \\ 2 & 1 & 0 \\ -1 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ -1 & 0 \\ 2 & -1 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 0 & -1 \end{bmatrix}$

   a) Find the matrix $D = A - (BC)^T$ **[6 marks]**

   b) Show that $|AD| = |DA|$ **[6 marks]**

Final Answers

1. a) $\frac{43}{29} - \frac{6}{29}i$

   b) $x = 0$

2. $\left(-\frac{5}{3}, 0\right) \cup (1, \infty)$

3. $T_{10} = \frac{-3}{32}$, $S_\infty = 32$

4. $\frac{1}{27} + \frac{1}{27}x + \frac{2}{81}x^2 + \frac{10}{729}x^3$, $(-3, 3)$, 0.0410

5. a) $x = 3$ $x = \frac{1}{81}$

   b) $x = \frac{1}{2}$

6. a) $D = \begin{bmatrix} 0 & 2 & -3 \\ 3 & 0 & 2 \\ -2 & 1 & -1 \end{bmatrix}$

Answer Scheme

1. a) $\dfrac{4+7i}{2+5i} = \dfrac{4+7i}{2+5i} \times \dfrac{2-5i}{2-5i} = \dfrac{43-6i}{29} = \dfrac{43}{29} - \dfrac{6}{29}i$

b) $5^{2x} + 4(5^x) - 5 = 0$

Let $y = 5^x$

$y^2 + 4y - 5 = 0$

$(y+5)(y-1) = 0$

$y = -5$ or $y = 1$

$5^x = -5$ (undefined)

$5^x = 1$

$5^x = 5^0$

$x = 0$

2. $\dfrac{5}{x} < 2 + 3x$

$\dfrac{5}{x} - 2 - 3x < 0$

$\dfrac{5 - 2x - 3x^2}{x} < 0$

$\dfrac{3x^2 + 2x - 5}{x} > 0$

$\dfrac{(x-1)(3x+5)}{x} > 0$

Let $x - 1 > 0$, $3x + 5 > 0$, $x > 0$

$x > 1$, $x > \dfrac{-5}{3}$, $x > 0$

By using number line: $\left(-\dfrac{5}{3}, 0\right) \cup (1, \infty)$

3. $S_4 = \dfrac{a\left[1-\left(-\frac{1}{2}\right)^4\right]}{1-\left(-\frac{1}{2}\right)}$

$30 = a\left(\frac{5}{8}\right)$

$a = 48$

$T_{10} = 48\left(-\frac{1}{2}\right)^9 = -\frac{3}{32}$

$S_\infty = \dfrac{48}{1-\left(-\frac{1}{2}\right)} = 32$

4. $\dfrac{1}{(3-x)^3} = (3-x)^{-3}$

$= 3^{-3}\left(1-\frac{x}{3}\right)^{-3}$

$= \frac{1}{27}\left[1+(-3)\left(-\frac{x}{3}\right)+\frac{(-3)(-4)}{2!}\left(-\frac{x}{3}\right)^2+\frac{(-3)(-4)(-5)}{3!}\left(-\frac{x}{3}\right)^3+\ldots\right]$

$= \frac{1}{27}+\frac{1}{27}x+\frac{2}{81}x^2+\frac{10}{729}x^3+\ldots$

For expansion valid

$\left|-\frac{x}{3}\right| < 1 \quad \Rightarrow \; -3 < x < 3$

The interval is $(-3\ ,\ 3)$

when $x = 0.1$

$\dfrac{1}{(3-0.1)^3} = \frac{1}{27}+\frac{1}{27}(0.1)+\frac{2}{81}(0.1)^2+\frac{10}{729}(0.1)^3+\ldots$

$\dfrac{1}{(2.9)^3} = 0.0410$ (4 d.p.)

5. a) $\log_3 x - \dfrac{4}{\log_3 x} + 3 = 0$

$(\log_3 x)^2 - 4 + 3\log_3 x = 0$

Substituting y for $\log_3 x$ gives :

$y^2 + 3y - 4 = 0$

$(y+4)(y-1) = 0$

$y = -4 \text{ or } 1$

$\log_3 x = -4 \quad \text{or} \quad \log_3 x = 1$

$x = 3^{-4} \quad \text{or} \quad x = 3$

$x = \dfrac{1}{81}$

NOTE: Check whether both values satisfy the original equation.

b) $\sqrt{3x+1} - \sqrt{2x-1} = \sqrt{x+2}$

$(\sqrt{3x+1} - \sqrt{2x-1})^2 = (\sqrt{x+2})^2$

$3x+1-2\sqrt{(3x+1)(2x-1)} + (2x-1) = x+2$

$-2\sqrt{(3x+1)(2x-1)} = -4x+2$

$\sqrt{(3x+1)(2x-1)} = 2x-1$

$(\sqrt{(3x+1)(2x-1)})^2 = (2x-1)^2$

$6x^2 - x - 1 = 4x^2 - 4x + 1$

$2x^2 + 3x - 2 = 0$

$(2x-1)(x+2) = 0$

$x = \dfrac{1}{2} \text{ or } x = -2 \qquad \text{but} \quad x \neq -2$

$\therefore x = \dfrac{1}{2}$

6. a) $BC = \begin{bmatrix} 1 & -1 \\ -1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 0 \\ 3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 1 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}$

$(BC)^T = \begin{bmatrix} -1 & -2 & 1 \\ -1 & 1 & -2 \\ 1 & 0 & 1 \end{bmatrix}$

$D = A - (BC)^T$

$= \begin{bmatrix} -1 & 0 & -2 \\ 2 & 1 & 0 \\ -1 & 1 & 0 \end{bmatrix} - \begin{bmatrix} -1 & -2 & 1 \\ -1 & 1 & -2 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 & -3 \\ 3 & 0 & 2 \\ -2 & 1 & -1 \end{bmatrix}$

b) $AD = \begin{bmatrix} 4 & -4 & 5 \\ 3 & 4 & -4 \\ 3 & -2 & 5 \end{bmatrix}$ $DA = \begin{bmatrix} 7 & -1 & 0 \\ -5 & 2 & -6 \\ 5 & 0 & 4 \end{bmatrix}$

$|AD| = 4(20-8) - (-4)(15+12) + 5(-6-12)$

$= 66$

$|DA| = 7(8-0) - (-1)(-20+30) + 0$

$= 66$

$\therefore |AD| = |DA| \; |AD| = 66$