1. a) Solve the equation $(x+yi)(3-i)=1+2i$ where x and y are real numbers.

[3 marks]

b) Simplify $\dfrac{3-2\sqrt{10}}{3+\sqrt{10}}+\dfrac{3+2\sqrt{10}}{3-\sqrt{10}}$. **[4 marks]**

2. Solve $x^5e^{-3\ln x}+4x=21$. **[6 marks]**

3. Solve the inequalities $2|5+x|>|x|$. **[6 marks]**

4. Given that the sum of the first n terms of an arithmetic series is $S_n=2n^2+3n$, find the sum of the first $(n-1)$ terms.
Hence, find the first term and the common difference. **[7 marks]**

5. Expand $\sqrt{4+3x}$ in ascending powers of x until and including term in x^3. State the range of x for which the expansion is valid. By substituting a suitable value of x, estimate the value of $\sqrt{19}$ correct to three decimal places.

[12 marks]

6. a) If $A=\begin{bmatrix}4&1&5\\3&2&7\\1&3&6\end{bmatrix}$, find the minor and cofactor of each element in the first column. Hence, find the determinant of A. **[6 marks]**

b) Find AB if $A=\begin{bmatrix}2&1&-3\\1&2&-4\\3&1&-2\end{bmatrix}$ and $B=\begin{bmatrix}0&-1&2\\-10&5&5\\-5&1&3\end{bmatrix}$. Hence, find the values of p, q and r if $A\begin{bmatrix}p\\q\\r\end{bmatrix}=\begin{bmatrix}3\\-2\\9\end{bmatrix}$. **[6 marks]**

Final Answers

1. a) $x = \frac{1}{10}, y = \frac{7}{10}$

   b) -58

2. $x = 3$

3. $(-\infty, -10) \cup (-\frac{10}{3}, \infty)$

4. $2n^2 - n - 1,\ T_1 = 5,\ d = 4$

5. $2 + \frac{3}{4}x - \frac{9}{64}x^2 + \frac{27}{512}x^3$; $-\frac{4}{3} < x < \frac{4}{3}$; 4.359

6. a) $M_{11} = -9,\ M_{21} = -9,\ M_{31} = -3$; $C_{11} = -9,\ C_{21} = 9,\ C_{31} = -3$; $|A| = -12$

   b) $AB = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$; $p = 4,\ q = 1,\ r = 2$

Answer Scheme

1. a)

$$(x+yi)(3-i)=1+2i$$
$$(x+yi)=\frac{1+2i}{3-i}$$
$$(x+yi)=\frac{1+2i}{3-i}\times\frac{3+i}{3+i}$$
$$(x+yi)=\frac{3+i+6i+2i^2}{9+1}$$
$$(x+yi)=\frac{3+i+6i-2}{9+1}$$
$$(x+yi)=\frac{1+7i}{10}$$
$$x=\frac{1}{7},\ y=\frac{7}{10}$$

b)

$$\frac{3-2\sqrt{10}}{3+\sqrt{10}}+\frac{3+2\sqrt{10}}{3-\sqrt{10}}=\frac{(3-2\sqrt{10})(3-\sqrt{10})+(3+2\sqrt{10})(3+\sqrt{10})}{(3+\sqrt{10})(3-\sqrt{10})}$$
$$=\frac{9-3\sqrt{10}-6\sqrt{10}+20+9+3\sqrt{10}+6\sqrt{10}+20}{9-10}$$
$$=\frac{58}{-1}$$
$$=-58$$

2.
$$x^5e^{-3\ln x}+4x=21$$
$$x^5e^{\ln x^{-3}}+4x=21$$
$$x^5(x^{-3})+4x=21$$
$$x^2+4x-21=0$$
$$(x+7)(x-3)=0$$
$x=-7$ or $x=3$

Since $x>0$, $x\neq -7$

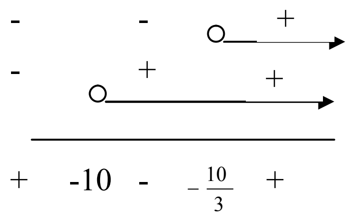
$x=3$

3.
$$2|5+x|>|x|$$
$$|10+2x|>|x|$$
$$(10+2x)^2>x^2$$
$$100+40x+4x^2>x^2$$
$$3x^2+40x+100>0$$

$(3x+10)(x+10)>0$

Let $3x+10>0 \Rightarrow x>-\dfrac{10}{3}$

$x+10>0 \Rightarrow x>-10$



Solution interval is $(-\infty, -10) \cup (-\dfrac{10}{3}, \infty)$

4. $S_n = 2n^2 + 3n$

$S_{n-1} = 2(n-1)^2 + 3(n-1)$

$S_{n-1} = 2(n^2 - 2n + 1) + 3n - 3$

$S_{n-1} = 2n^2 - n - 1$

$T_1 = S_1 = 2(1)^2 + 3(1) = 5$

$d = T_2 - T_1$

$d = (S_2 - S_1) - T_1$

$d = [(2(2)^2 + 3(2)) - 5] - (5) = 4$

5. $\sqrt{4+3x} = \left[4\left(1+\frac{3}{4}x\right)\right]^{\frac{1}{2}}$

$= 4^{\frac{1}{2}}\left(1+\frac{3}{4}x\right)^{\frac{1}{2}}$

$= 2\left[1+\frac{1}{2}\left(\frac{3}{4}x\right)+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}\left(\frac{3}{4}x\right)^2+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\left(\frac{3}{4}x\right)^3\right]$

$= 2 + \frac{3}{4}x - \frac{9}{64}x^2 + \frac{27}{512}x^3$

For expansion valid

$$\left|\frac{3}{4}x\right| < 1$$

$$|x| < \frac{4}{3}$$

$$-\frac{4}{3} < x < \frac{4}{3}$$

$$x = \frac{1}{4}\ ,$$

$$\sqrt{4+3\left(\frac{1}{4}\right)} = 2 + \frac{3}{4}\left(\frac{1}{4}\right) - \frac{9}{64}\left(\frac{1}{4}\right)^2 + \frac{27}{512}\left(\frac{1}{4}\right)^3$$

$$\frac{\sqrt{19}}{2} = 2 + \frac{3}{4}\left(\frac{1}{4}\right) - \frac{9}{64}\left(\frac{1}{4}\right)^2 + \frac{27}{512}\left(\frac{1}{4}\right)^3$$

$$\sqrt{19} = 4.359$$

6. a) $A = \begin{bmatrix} 4 & 1 & 5 \\ 3 & 2 & 7 \\ 1 & 3 & 6 \end{bmatrix}$

$$M_{11} = \begin{vmatrix} 2 & 7 \\ 3 & 6 \end{vmatrix} = -9$$

$$M_{21} = \begin{vmatrix} 1 & 5 \\ 3 & 6 \end{vmatrix} = -9$$

$$M_{31} = \begin{vmatrix} 1 & 5 \\ 2 & 7 \end{vmatrix} = -3$$

$$C_{11} = M_{11} = -9$$

$$C_{21} = -M_{11} = 9$$

$$C_{31} = M_{31} = -3$$

$$|A| = a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31}$$

$$= 4(-9) + 3(9) + 1(-3)$$

$$= -12$$

b) $A = \begin{bmatrix} 2 & 1 & -3 \\ 1 & 2 & -4 \\ 3 & 1 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 & 2 \\ -10 & 5 & 5 \\ -5 & 1 & 3 \end{bmatrix}$

$$AB = \begin{bmatrix} 2 & 1 & -3 \\ 1 & 2 & -4 \\ 3 & 1 & -2 \end{bmatrix}\begin{bmatrix} 0 & -1 & 2 \\ -10 & 5 & 5 \\ -5 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$AB = 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 5I$$

$$A\left(\frac{1}{5}B\right) = I$$

$$A^{-1} = \frac{1}{5}B$$

$$A\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 9 \end{bmatrix}$$

$$AX = b$$

$$X = A^{-1}b$$

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \frac{1}{5}Bb = \frac{1}{5}\begin{bmatrix} 0 & -1 & 2 \\ -10 & 5 & 5 \\ -5 & 1 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \\ 9 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$p = 4$, $q = 1$, $r = 2$