1. Find the polar form of the complex number $z=\dfrac{2-5i}{i}$ **[7 marks]**

2. Solve the inequalities $\dfrac{1+x}{1-x}<1$ **[7 marks]**

3. a) Simplify $\dfrac{2+\sqrt{5}}{3+\sqrt{5}}+\sqrt{5}$ **[3 marks]**

   b) Solve $7\log_x 3-\log_3 x=6$ **[5 marks]**

4. The sum of the first 8 terms of an arithmetic sequence is 60 and the sum of the next 6 terms is 108. Find the 25th term of this arithmetic sequence.

   [7 marks]

5. By using binomial expansion, expand $\dfrac{1}{(2-x)^2}$ in ascending powers of x until the term x^3. Hence,

   a) state the range values of x for the expansion to be valid.

   b) find the value of $(1.9)^{-2}$ correct to three decimal places. **[10 marks]**

6. Given $P=\begin{bmatrix}3 & 2 & 5\\ n & 3 & 7\\ 2 & 1 & 5\end{bmatrix}$ and the minor of $a_{32}=1$.

   a) show that $n=4$ **[3 marks]**

   b) Find $|P|$ and adj P. Hence, find P^{-1}. **[9 marks]**

Final Answers

1. $z=\sqrt{29}\left[\cos\left(-2.761+i\sin(-2.761)\right)\right]$

2. $\left(-\infty,0\right)\cup\left(1,\infty\right)$

3. a) $\dfrac{1+5\sqrt{5}}{4}$

   b) $x=3$, $\dfrac{1}{2187}$

4. $T_{25}=\dfrac{153}{4}$

5. $\dfrac{1}{4}+\dfrac{1}{4}x+\dfrac{3}{16}x^{2}+\dfrac{1}{8}x^{3}+....$

   a) $-2<x<2$

   b) 0.277

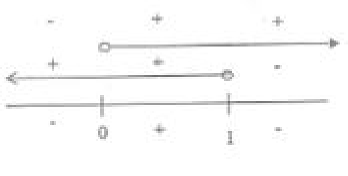
6. $\left|P\right|=2$, $\operatorname{adj}P=\begin{bmatrix}8 & -5 & -1\\ -6 & 5 & -1\\ -2 & 1 & 1\end{bmatrix}$, $P^{-1}=\dfrac{1}{2}\begin{bmatrix}8 & -5 & -1\\ -6 & 5 & -1\\ -2 & 1 & 1\end{bmatrix}$

Answer Scheme

1. $$Z = \frac{2-5i}{i} \times \left(\frac{i}{i}\right)$$
$$= \frac{5+2i}{i^2}$$
$$= \frac{5+2i}{-1}$$
$$z = -5-2i$$
$$|z| = \sqrt{(-5)^2 + (-2)^2}$$
$$= \sqrt{29}$$
$$\theta = -\left[\pi - \tan^{-1}\left(\frac{-2}{-5}\right)\right]$$
$$= -[\pi - 0.3805]\text{rad}$$
$$= -2.761 \text{ rad}$$
The polar form $z = \sqrt{29}[\cos(-2.761 + i\sin(-2.761))]$.

2.

$$\frac{1+x}{1-x} < 1$$
$$\frac{1+x}{1-x} - 1 < 0$$
$$\frac{2x}{1-x} < 0$$
$x > 0$ or $1 - x > 0$
$x < 1$



$(-\infty, 0) \cup (1, +\infty)$

3. a) $$\frac{2+\sqrt{5}}{3+\sqrt{5}} + \sqrt{5} = \left(\frac{2+\sqrt{5}}{3+\sqrt{5}}\right)\left(\frac{3-\sqrt{5}}{3-\sqrt{5}}\right) + \sqrt{5}$$

$$= \frac{1+\sqrt{5}}{4} + \sqrt{5}$$
$$= \frac{1+5\sqrt{5}}{4}$$

b) $7\log_x 3 - \log_3 x = 6$

$$7\frac{\log_3 3}{\log_3 x} - \log_3 x = 6$$

$$\frac{7}{\log_3 x} - \log_3 x = 6$$

Let $\log_3 x = a$

$$\frac{7}{a} - a = 6$$

$$7 - a^2 = 6a$$

$$7 - a^2 - 6a = 0$$

$$a^2 + 6a - 7 = 0$$

$$(a-1)(a+7) = 0$$

$$a = 1, \quad a = -7$$

$\log_3 x = 1,$ $\quad$ $\log_3 x = -7$

$x = 3^1$ $\quad$ $x = 3^{-7}$

4. $S_8 = 60$

$$\frac{8}{2}[2a+7d] = 60$$
$2a + 7d = 15$ ----------------(1)
$$S_{14} - S_8 = 108$$
$$\frac{14}{2}(2a+13d) - 60 = 108$$
$2a + 13d = 24$ -------------------(2)
$$6d = 9,\ d = \frac{3}{2}$$

Substitute into (1)

$a = \dfrac{9}{4}$

$T_{25} = \dfrac{9}{4} + 24(\dfrac{3}{2}) = \dfrac{153}{4}$

5. $\dfrac{1}{(2-x)^2} = (2-x)^{-2}$

$= 2^{-2}\left(1 - \dfrac{x}{2}\right)^{-2}$

$$\frac{1}{4}\left(1 + (-2)\left(-\frac{x}{2}\right) + \frac{(-2)(-3)}{2!}\left(-\frac{x}{2}\right)^2 + \frac{(-2)(-3)(-4)}{3!}\left(-\frac{x}{2}\right)^3\right) =$$

$$\frac{1}{4}\left(1 + x + \frac{3}{4}x^2 + \frac{1}{2}x^3 + ..\right)$$

$$= \frac{1}{4} + \frac{1}{4}x + \frac{3}{16}x^3 + \frac{1}{8}x^3 + ...$$

a) $\left|-\dfrac{1}{2}x\right| < 1$

$-1 < \dfrac{1}{2}x < 1$

$-2 < x < 2$

b) $(1.9)^{-2} = (2-0.1)^{-2}$

$= \dfrac{1}{4} + \dfrac{1}{4}(0.1) + \dfrac{3}{16}(0.1)^2 + \dfrac{1}{8}(0.1)^3 + ...$

$= 0.277$

6. $P = \begin{bmatrix} 3 & 2 & 5 \\ n & 3 & 7 \\ 2 & 1 & 5 \end{bmatrix}$

a) $M_{32} = \begin{vmatrix} 3 & 5 \\ n & 7 \end{vmatrix} = 3(7) - 5n = 1$

$21 - 5n = 1$

$5n = 20$

$n = 4$

b) $|P| = 3\begin{vmatrix} 3 & 7 \\ 1 & 5 \end{vmatrix} - 2\begin{vmatrix} 4 & 7 \\ 2 & 5 \end{vmatrix} + 5\begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix}$

$= 3(15-7) - 2(20-14) + 5(4-6)$

$= 2$

$$\text{adj } P = \begin{bmatrix} +\begin{vmatrix} 3 & 7 \\ 1 & 5 \end{vmatrix} & -\begin{vmatrix} 4 & 7 \\ 2 & 5 \end{vmatrix} & +\begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix} \\ -\begin{vmatrix} 2 & 5 \\ 1 & 5 \end{vmatrix} & +\begin{vmatrix} 3 & 5 \\ 2 & 5 \end{vmatrix} & -\begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} \\ +\begin{vmatrix} 2 & 5 \\ 3 & 7 \end{vmatrix} & -\begin{vmatrix} 3 & 5 \\ 4 & 7 \end{vmatrix} & +\begin{vmatrix} 3 & 2 \\ 4 & 3 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} 8 & -6 & -2 \\ -5 & 5 & 1 \\ -1 & -1 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 8 & -5 & -1 \\ -6 & 5 & -1 \\ -2 & 1 & 1 \end{bmatrix}$$

$$\therefore\ P^{-1} = \frac{1}{|P|} adj\ P$$

$$= \frac{1}{2}\begin{bmatrix} 8 & -5 & -1 \\ -6 & 5 & -1 \\ -2 & 1 & 1 \end{bmatrix}$$