1. Solve the equation $2^{2x-3} - 5(2^x) + 32 = 0$. **[5 marks]**

2. Solve the inequality $2(x+1) < x^2 - 6 < 4 - 3x$. **[6 marks]**

3. a) Given that $\log_x 9 = a^2$ and $\log_y 3 = b$. Express $\log_9 xy^2$ in terms of $a$ and $b$.
**[3 marks]**

b) If $z = 5 - 2i$, write the complex number $\dfrac{z+4i}{3-iz}$ in the Cartesian and polar form.
**[6 marks]**

4. In a geometric sequence, the sum of the first five terms is 44 and the sum of the next five terms is $-\dfrac{11}{8}$. Find the common ratio and first term of this series. Find also the sum to infinity of this series. **[7 marks]**

5. Expand $(1-x)^{\frac{1}{2}}$ in ascending powers of $x$ until and including the term in $x^3$. State the range of $x$ for which the expansion is valid. Hence, by substituting $x = \dfrac{1}{64}$, evaluate $\sqrt{7}$ correct to five decimal places. **[11 marks]**

6. a) The matrices $A$ and $B$ are given by

$$A = \begin{bmatrix} -1 & 2 & 1 \\ -3 & 1 & 4 \\ 0 & 1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} -35 & 19 & 18 \\ -27 & -13 & 45 \\ -3 & 12 & 5 \end{bmatrix}$$

Find the matrix $A^2B$ and deduce the inverse of $A$. **[6 marks]**

b) Hence, solve the system of linear equations

$$x - 2y - z = -8$$
$$3x - y - 4z = -15$$
$$y + 2z = 4$$

**[6 marks]**

**Final Answers**

1. $x = 3$ or $x = 5$

2. $(-5, -2)$

3. a) $\dfrac{1}{a^2} + \dfrac{1}{b}$

   b) $-\dfrac{5}{26} + \dfrac{27}{26}i$ ; $\dfrac{\sqrt{754}}{26}\left(\cos(1.75) + i\sin(1.75)\right)$

4. $r = -\dfrac{1}{2}$ , $a = 64$ , $S_\infty = \dfrac{128}{3}$

5. $1 - \dfrac{1}{2}x - \dfrac{1}{8}x^2 - \dfrac{1}{16}x^3$ ; $-1 < x < 1$ ; 2.64575

6. a) $A^2B = \begin{bmatrix} 121 & 0 & 0 \\ 0 & 121 & 0 \\ 0 & 0 & 121 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} -\dfrac{2}{11} & -\dfrac{3}{11} & \dfrac{7}{11} \\ \dfrac{6}{11} & -\dfrac{2}{11} & \dfrac{1}{11} \\ -\dfrac{3}{11} & \dfrac{1}{11} & \dfrac{5}{11} \end{bmatrix}$

   b) $x = -3$, $y = 2$, $z = 1$

**Answer Scheme**

1. $2^{2x-3} - 5(2^x) + 32 = 0$

$2^{2x}(2)^{-3} - 5(2^x) + 32 = 0$

Let $y = 2^x$

$\frac{1}{8}y^2 - 5y + 32 = 0$

$y^2 - 40y + 256 = 0$

$(y-8)(y-32) = 0$

$y = 8$ or $y = 32$

$2^x = 2^3$ or $2^x = 2^5$

$x = 3$ or $x = 5$

2. $2(x+1) < x^2 - 6 < 4 - 3x$

$2(x+1) < x^2 - 6$ and $x^2 - 6 < 4 - 3x$

$2x + 2 < x^2 - 6$

$x^2 - 2x - 8 > 0$

$(x+2)(x-4) > 0$

Let $x + 2 > 0 \Rightarrow x > -2$

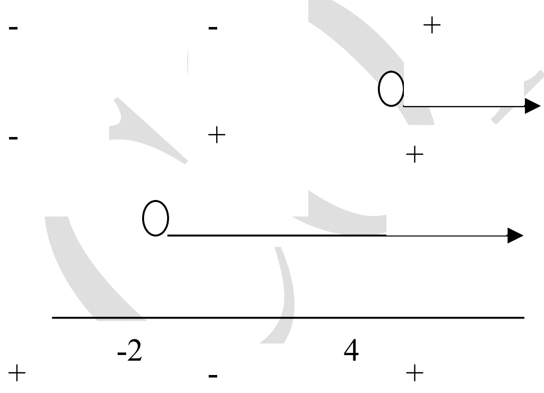
$x - 4 > 0 \Rightarrow x > 4$

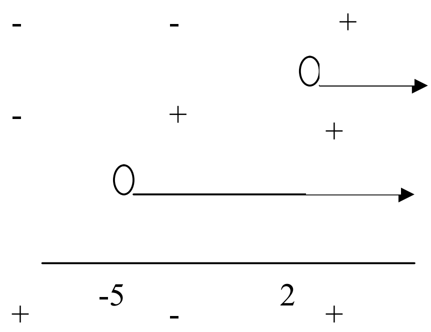
$x^2 + 3x - 10 < 0$

$(x+5)(x-2) < 0$

Let $x + 5 > 0 \Rightarrow x > -5$

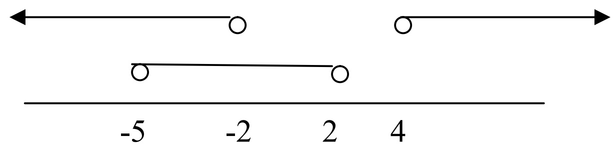
$x - 2 > 0 \Rightarrow x > 2$




$(-\infty, -2) \cup (4, \infty)$

$(-5, 2)$




Solution interval is $( -5 \, , -2 )$

3. a) $\log_x 9 = a^2$ and $\log_y 3 = b$

$$\log_9 xy^2 = \log_9 x + 2\log_9 y$$
$$= \frac{1}{\log_x 9} + \frac{2}{\log_y 9}$$
$$= \frac{1}{\log_x 9} + \frac{2}{2\log_y 3}$$
$$= \frac{1}{a^2} + \frac{1}{b}$$

b) $z = 5 - 2i$

$$\frac{z+4i}{3-iz} = \frac{(5-2i)+4i}{3-i(5-2i)}$$
$$= \frac{5+2i}{3-5i+2i^2}$$
$$= \frac{5+2i}{1-5i}\,\frac{1+5i}{1+5i}$$
$$= \frac{5+25i+2i+10i^2}{1+25}$$
$$= -\frac{5}{26} + \frac{27}{26}i$$

$$\left| -\frac{5}{26} + \frac{27}{26}i \right| = \sqrt{\left(\frac{-5}{26}\right)^2 + \left(\frac{27}{26}\right)^2} = \frac{\sqrt{754}}{26}$$

$$Arg, \theta \quad = \pi - \tan^{-1}\left(\frac{27}{5}\right) = 1.75$$

Polar form : $\dfrac{\sqrt{754}}{26}\left(\cos(1.75) + i\sin(1.75)\right)$

4. $S_5 = 44$

$$\frac{a(1-r^5)}{1-r} = 44 \qquad (1)$$

$$S_{10} - S_5 = -\frac{11}{8}$$

$$S_{10} - 44 = -\frac{11}{8}$$

$$S_{10} = \frac{341}{8}$$

$$\frac{a(1-r^{10})}{1-r} = \frac{341}{8}$$

$$\frac{a(1+r^5)(1-r^5)}{1-r} = \frac{341}{8} \qquad (2)$$

$(2) \div (1)$ :

$$1 + r^5 = \frac{31}{32}$$

$$r^5 = -\frac{1}{32}$$

$$r = -\frac{1}{2}$$

$$(1) : \frac{a\left[1-\left(-\frac{1}{2}\right)^5\right]}{1-\left(-\frac{1}{2}\right)} = 44$$

$$a = 64$$

$$S_\infty = \frac{64}{1-\left(-\frac{1}{2}\right)}$$

$$= \frac{128}{3}$$

5. $$(1-x)^{\frac{1}{2}} = 1 + \frac{1}{2}(-x) + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}(-x)^2 + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}(-x)^3$$

$$= 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3$$

For expansion valid

$$|-x| < 1$$

$$-1 < x < 1$$

$$x = \frac{1}{64} \ ,$$

$$\left(1-\frac{1}{64}\right)^{\frac{1}{2}}=1-\frac{1}{2}\left(\frac{1}{64}\right)-\frac{1}{8}\left(\frac{1}{64}\right)^2-\frac{1}{16}\left(\frac{1}{64}\right)^3$$

$$\left(\frac{63}{64}\right)^{\frac{1}{2}}=1-\frac{1}{2}\left(\frac{1}{64}\right)-\frac{1}{8}\left(\frac{1}{64}\right)^2-\frac{1}{16}\left(\frac{1}{64}\right)^3$$

$$\frac{3}{8}(7)^{\frac{1}{2}}=1-\frac{1}{2}\left(\frac{1}{64}\right)-\frac{1}{8}\left(\frac{1}{64}\right)^2-\frac{1}{16}\left(\frac{1}{64}\right)^3$$

$$\sqrt{7}=2.64575 \qquad \text{(5 d.p.)}$$

6. a) $A^2B = AAB$

$$=\begin{bmatrix}-1 & 2 & 1\\ -3 & 1 & 4\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}-1 & 2 & 1\\ -3 & 1 & 4\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}-35 & 19 & 18\\ -27 & -13 & 45\\ -3 & 12 & 5\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 2 & 1\\ -3 & 1 & 4\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}-22 & -33 & 77\\ 66 & -22 & 11\\ -33 & 11 & 55\end{bmatrix}$$

$$A^2B=\begin{bmatrix}121 & 0 & 0\\ 0 & 121 & 0\\ 0 & 0 & 121\end{bmatrix}$$

$$A^2B=121\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$AAB=121I$$

$$A\left(\frac{1}{121}AB\right)=I$$

$$A^{-1}=\frac{1}{121}AB$$

$$=\frac{1}{121}\begin{bmatrix}-22 & -33 & 77\\ 66 & -22 & 11\\ -33 & 11 & 55\end{bmatrix}$$

$$A^{-1}=\begin{bmatrix}-\frac{2}{11} & -\frac{3}{11} & \frac{7}{11}\\ \frac{6}{11} & -\frac{2}{11} & \frac{1}{11}\\ -\frac{3}{11} & \frac{1}{11} & \frac{5}{11}\end{bmatrix}$$

b) $$x-2y-z=-8$$
$$3x-y-4z=-15$$
$$y+2z=4$$

$$-x+2y+z=8$$
$$-3x+y+4z=15$$
$$y+2z=4$$
$$\begin{bmatrix}-1 & 2 & 1\\ -3 & 1 & 4\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}8\\ 15\\ 4\end{bmatrix}$$
$$AX=C$$
$$A^{-1}(AX)=A^{-1}C$$
$$X=\begin{bmatrix}-\frac{2}{11} & -\frac{3}{11} & \frac{7}{11}\\ \frac{6}{11} & -\frac{2}{11} & \frac{1}{11}\\ -\frac{3}{11} & \frac{1}{11} & \frac{5}{11}\end{bmatrix}\begin{bmatrix}8\\ 15\\ 4\end{bmatrix}$$
$$\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}-3\\ 2\\ 1\end{bmatrix}$$
$$x=-3,\ y=2,\ z=1$$