1. Solve the equation $e^{2x} - 3e^{x} + 2 = 0$ **[6 marks]**

2. The sum of the first ten terms of an arithmetic series is 455 and the seventh term is twice of the third term. Find the first term and the common difference of the series. **[7 marks]**

3. Given $z_1 = 3 - 5i$ and $z_2 = 2 + 6i$. Find $\frac{\overline{z_1}}{z_2}$ and write the answer in the form of $a + bi$. Hence, express in polar form. **[7 marks]**

4. Determine the solution set of the inequality $\left|\frac{x}{5+x}\right| > 2$. **[7 marks]**

5. Expand $(1+2x)^{1/3}$ in ascending powers of x up to the term x^2 and determine the range of x such that this expansion is valid. Hence, by substituting $x = \frac{5}{54}$ show that $\sqrt[3]{4} \approx \frac{749}{486}$. **[11 marks]**

6. a) Given the matrix $A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{pmatrix}$. Find the cofactor, determinant and A^{-1} of matrix *A*. **[7 marks]**

   b) Consider the following system of linear equation:

$$\begin{aligned} 6x + 6y + 3z &= 48 \\ 3x + 3y + 3z &= 27 \\ 6x + 3y + 3z &= 39 \end{aligned}$$

   i) Write the system of linear equation in the form of a matrix equation $kAX = B$ where *k* is a constant and $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$. **[2 marks]**

   ii) Hence, find the values of *x*, *y* and *z*. **[3 marks]**

Final Answers

1. $x = 0 \;\; or \;\; x = \ln 2$
2. $a = 14$ and $d = 7$
3. $-\frac{3}{5} + \frac{7}{10}i$

   polar form: $\sqrt{\frac{17}{20}}\left[\cos(2.279) + \sin(2.279)i\right]$
4. $\left\{x : -10 < x < -\frac{10}{3}, x \neq -5\right\}$
5. $1 + \frac{2}{3}x - 4x^2 + ...$, valid for $|2x| < 1 \;\; or \;\; -\frac{1}{2} < x < \frac{1}{2}$
6. a) $A^{-1} = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 2 & 0 \end{pmatrix}$

   b) ii) $x = 4$, $y = 3$ and $z = 2$

Answer Scheme

1. $e^{2x} - 3e^{x} + 2 = 0$

Let $a = e^{x}$

$a^2 - 3a + 2 = 0$

$(a-2)(a-1) = 0$

$a = 2, \quad a = 1$

$e^{x} = 2, \qquad e^{x} = 1$
$x = \ln 2, \qquad x = \ln 1 = 0$

2. $S_{10} = 455$

$\frac{10}{2}(2a + 9d) = 455$

$2a + 9d = 91$ ……… Eq(1)

$T_7 = 2T_3$

$a + 6d = 2(a + 2d)$

$a = 2d$ …………… Eq(2)

Substitute (2) into (1): $2(2d) + 9d = 91 \Rightarrow d = 7$ into (2)
$a = 2(7) = 14$

3. $\frac{\overline{Z_1}}{Z_2} = \frac{3+5i}{2-6i} \times \frac{2+6i}{2+6i}$

$= \frac{6 + 18i + 10i + 30i^2}{40}$

$= \frac{-24}{40} + \frac{28}{40}i$ @ $-\frac{3}{5} + \frac{7}{10}i$

$|Z| = \sqrt{\left(-\frac{3}{5}\right)^2 + \left(\frac{7}{10}\right)^2} \quad = \sqrt{\frac{17}{20}}$

$Arg(z), \alpha = \tan^{-1}\left|\frac{\frac{7}{10}}{\frac{3}{5}}\right|$

$= \tan^{-1}\left|\frac{7}{6}\right| = 0.862\, rad$

$= \tan^{-1}\left|\frac{7}{6}\right| = 0.862\, rad$

$\theta = \pi - 0.862 = 2.279\, rad$

Polar form: $= \sqrt{\frac{17}{20}}\left[\cos(2.279) + \sin(2.279)i\right]$

4. $$\left|\frac{x}{5+x}\right| > 2$$

$$\frac{x}{5+x} > 2 \qquad \text{OR} \qquad \frac{x}{5+x} < -2$$

$$\frac{x-2(5+x)}{5+x} > 0 \qquad\qquad \frac{x+2(5+x)}{5+x} < 0$$

$$\frac{-x-10}{5+x} > 0 \qquad\qquad \frac{3x+10}{5+x} < 0$$

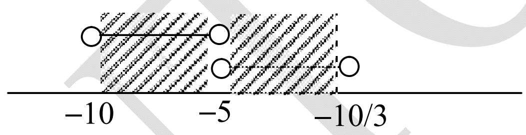
$$x=-10\ ,\ x=-5 \qquad\qquad x=-\frac{10}{3}\ ,\ x=-5$$

x	$(-\infty,-10)$	$(-10,-5)$	$(-5,\infty)$
$-x-10$	+	–	–
$5+x$	–	–	+
$\frac{-x-10}{5+x}$	–	+	–

x	$(-\infty,-5)$	$\left(-5,-\frac{10}{3}\right)$	$\left(-\frac{10}{3},\infty\right)$
$3x+10$	–	–	+
$5+x$	–	+	+
$\frac{3x+10}{5+x}$	+	–	+

$(-10,-5)$

$\left(-5,-\frac{10}{3}\right)$



$$\therefore \left\{x : -10 < x < -\frac{10}{3}, x \neq -5\right\}$$

5.

Solution

$$(1+2x)^{\frac{1}{3}} = 1 + \frac{1}{3}(2x) + \frac{\frac{1}{3}(\frac{1}{3}-1)}{2!}(2x)^2 + \ldots$$

$$= 1 + \frac{2}{3}x - 4x^2 + \ldots$$

Substituting $x = \frac{5}{54}$

$$\left[1+2\left(\frac{5}{54}\right)\right]^{\frac{1}{3}} \approx 1 + \frac{2}{3}\left(\frac{5}{54}\right) - 4\left(\frac{5}{54}\right)^2$$

$$\left(\frac{64}{54}\right)^{\frac{1}{3}} \approx 1 + \frac{10}{162} - \frac{100}{2916}$$

$$\left(\frac{32}{27}\right)^{\frac{1}{3}} \approx \frac{749}{729}$$

$$\frac{2\sqrt[3]{4}}{3} = \frac{749}{729}$$

$$\therefore \quad \sqrt[3]{4} = \frac{3}{2} \times \frac{749}{729} \approx \frac{749}{486}$$

6. a)

$$c_{ij} = \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 2 \\ 1 & -1 & 0 \end{pmatrix}$$

$$|A| = 2(0) + 2(1) + 1(-1) = 1$$

$$A^{-1} = \frac{1}{1}\begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 2 \\ 1 & -1 & 0 \end{pmatrix}^T = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 2 & 0 \end{pmatrix}$$

b) i)

$$\begin{pmatrix} 6 & 6 & 3 \\ 3 & 3 & 3 \\ 6 & 3 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 48 \\ 27 \\ 39 \end{pmatrix} \Rightarrow 3\begin{pmatrix} 2 & 2 & 1 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 48 \\ 27 \\ 39 \end{pmatrix} \Rightarrow 3AX = B$$

ii)

$$3AX = B$$

$$X = \frac{1}{3}A^{-1}B$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 2 & 0 \end{pmatrix}\begin{pmatrix} 48 \\ 27 \\ 39 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix}$$

Therefore, $x = 4$, $y = 3$ and $z = 2$