1. a) Simplify $\dfrac{2\sqrt{5}}{\sqrt{3}-\sqrt{2}}$. **[3 marks]**

   b) Solve the equation $\log_2(x^2+2)=1+\log_2(x+5)$. **[4 marks]**

2. Find the solution set of the inequalities $x+|3x+4|\le 6$ **[5 marks]**

3. Given the complex number z and its conjugate $\bar{z}$ satisfy the equation $z\bar{z}+2zi=12+6i$.

   Find z. **[7 marks]**

4. Solve the equations of $\sqrt{8x+17}-\sqrt{2x}=\sqrt{2x+9}$. **[7 marks]**

5. a) If the second term and the fifth term of the geometric progression are $\dfrac{1}{4}$ and $\dfrac{1}{32}$ respectively, find the first term and the common ratio. **[5 marks]**

   b) Expand $(1+3x)^{\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^3.

   State the range of values of x for which the expansion is valid. Hence, evaluate $\sqrt[3]{1.03}$ correct to four decimal places. **[7 marks]**

6. Given that $A=\begin{pmatrix}4 & 1 & 2\\ -2 & 3 & 1\\ -1 & 3 & 1\end{pmatrix}$ and $B=\begin{pmatrix}0 & -5 & 5\\ -1 & -6 & 8\\ 3 & 13 & -14\end{pmatrix}$

   a) Find AB as a single matrix.

   b) By using the result in (a), solve the following system linear equations.

   $$4x+y+2z=13$$

   $$-2x+3y+z=-9$$

   $$-x+3y+z=-7$$

   [12 marks]

CONTOH

Final Answers

1. a) $2\sqrt{15}+2\sqrt{10}$
   b) $x=4$ or -2

2. $\{\boldsymbol{x} : -5 \le \boldsymbol{x} \le \frac{1}{2}\}$

3. $z=3+3i$, $z=3-i$

4. $x=8$

5. a) $r=\frac{1}{2}$, $a=\frac{1}{2}$

   b) $1+x-x^2+\frac{5}{3}x^3+.....$,the expansion is valid if $-\frac{1}{3}<x<\frac{1}{3}$, 1.0099 (4 dp)

6. a) $5I$
   b) $x=2$, $y=-3$, $z=4$

CONTOH

Answer Scheme

1. a) $\dfrac{2\sqrt{5}}{\sqrt{3}-\sqrt{2}} = \dfrac{2\sqrt{5}}{\sqrt{3}-\sqrt{2}} \times \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$$= \frac{2\sqrt{5}\left(\sqrt{3}+\sqrt{2}\right)}{3-2}$$

$$= 2\sqrt{5}(\sqrt{3}+\sqrt{2})$$

$$= 2\sqrt{15} + 2\sqrt{10}$$

b) $\log_2(x^2+2) - \log_2(x+5) = 1$

$$\log_2\left(\frac{x^2+2}{x+5}\right) = 1$$

$$\frac{x^2+2}{x+5} = 2^1$$

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

$$x = 4 \text{ or } -2$$

2. $x + |3x+4| \le 6$

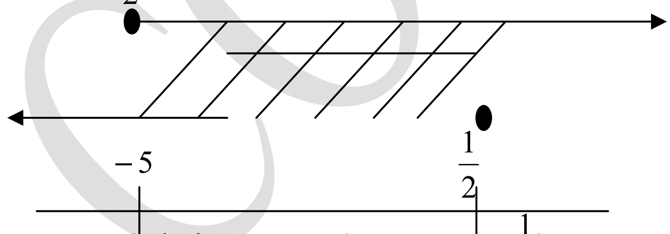
$|3\boldsymbol{x}+4| \le 6 - \boldsymbol{x}$

Using basic definition,

$3\boldsymbol{x}+4 \le 6-\boldsymbol{x}$ AND $3\boldsymbol{x}+4 \ge -(6-\boldsymbol{x})$

$4\boldsymbol{x} \le 2$ $\qquad 2\boldsymbol{x} \ge -10$

$\boldsymbol{x} \le \dfrac{1}{2}$ $\qquad \boldsymbol{x} \ge -5$



Solution set $\{\boldsymbol{x} : -5 \le \boldsymbol{x} \le \frac{1}{2}\}$

CONTOH

3. Let $z = x + iy$, $z = x - iy$

$z\bar{z} + 2zi = 12 + 6i$
$(x+iy)(x-iy) + 2i(x+iy) = 12 + 6i$
$x^2 - y^2 i^2 + 2xi + 2i^2 y = 12 + 6i$
$x^2 + y^2 - 2y + 2xi = 12 + 6i$

Comparing real parts
$x^2 + y^2 - 2y = 12$ -----------(1)

compare imaginary parts
$2x = 6$, $x = 3$ ------(2)

Substitute (2) into (1)
$(3)^2 + y^2 - 2y = 12$
$y^2 - 2y - 3 = 0$
$(y-3)(y+1) = 0$
$y = 3$ or -1

$z = 3 + 3i$, $z = 3 - i$

4. $\sqrt{8x+17} - \sqrt{2x} = \sqrt{2x+9}$
Squaring both sides
$8x + 17 - 2\left(\sqrt{8x+17}\right)\left(\sqrt{2x}\right) + 2x = 2x + 9$
$8x + 8 = 2\sqrt{(8x+17)(2x)}$
$4x + 4 = \sqrt{16x^2 + 34x}$
$16x^2 + 32x + 16 = 16x^2 + 34x$
$2x = 16$, $x = 8$.

5. a) $T_2 = ar = \frac{1}{4}$ ………(1) , $T_5 = ar^4 = \frac{1}{32}$ ……….(2)

(2) ÷ (1) , $\frac{ar^4}{ar} = \frac{1}{32} \div \frac{1}{4}$

$r^3 = \frac{1}{8}$

$r = \frac{1}{2}$

By substituting $r = \frac{1}{2}$ into (1),

$\frac{1}{2}a = \frac{1}{4}$

$a = \frac{1}{2}$

CONTOH

b) $(1+3x)^{\frac{1}{3}} = 1+\frac{1}{3}(3x)+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}(3x)^2+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)}{3!}(3x)^3+.....$

$= 1+x-x^2+\frac{5}{3}x^3+.....$

The expansion is valid if $|3x|<1$

$$|x|<\frac{1}{3}$$

$$-\frac{1}{3}<x<\frac{1}{3}$$

$\sqrt[3]{1.03} = (1+0.03)^{\frac{1}{3}}$

$= (1+3(0.01))^{\frac{1}{3}}$

So, we can use the above expansion by substituting $x = 0.01$.

$\sqrt[3]{1.03} = 1+0.01-(0.01)^2+\frac{5}{3}(0.01)^3+....$

$=1+0.01-0.0001+0.00000167$

$=1.00990167$

= 1.0099 (4 decimal places)

6. a) $AB = \begin{pmatrix} 4 & 1 & 2 \\ -2 & 3 & 1 \\ -1 & 3 & 1 \end{pmatrix}\begin{pmatrix} 0 & -5 & 5 \\ -1 & -6 & 8 \\ 3 & 13 & -14 \end{pmatrix}$

$= 5\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} = 5I$

b) $AB = 5I$

$A^{-1} = \frac{1}{5}B$

$= \frac{1}{5}\begin{pmatrix} 0 & -5 & 5 \\ -1 & -6 & 8 \\ 3 & 13 & -14 \end{pmatrix}$

$\begin{pmatrix} 4 & 1 & 2 \\ -2 & 3 & 1 \\ -1 & 3 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 13 \\ -9 \\ -7 \end{pmatrix}$

CONTOH

$$AX = B$$
$$X = A^{-1}B$$
$$= \frac{1}{5}\begin{pmatrix} 0 & -5 & 5 \\ -1 & -6 & 8 \\ 3 & 13 & -14 \end{pmatrix}\begin{pmatrix} 13 \\ -9 \\ -7 \end{pmatrix}$$
$$= \frac{1}{5}\begin{pmatrix} 0+45-35 \\ -13+54-56 \\ 39-117+98 \end{pmatrix}$$
$$= \frac{1}{5}\begin{pmatrix} 10 \\ -15 \\ 20 \end{pmatrix}$$
$$= \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$$
$x = 2$, $y = -3$, $z = 4$

CONTOH