1. Given $z_1 = 1 + 2i$ and $z_2 = 3 - 2i$. Express $z = z_1 + \frac{1}{\bar{z}_2}$ in the form of $a + bi$ where $\bar{z}_2$ is the conjugate of z_2. Hence find modulus and argument of z. **[6 marks]**

2. Given the matrices $P = \begin{pmatrix} 3 & 6 & 3 \\ 3 & -3 & 0 \\ 9 & 3 & 3 \end{pmatrix}$ and $Q = \begin{pmatrix} -1 & -1 & 4 \\ -1 & -2 & 5 \\ 1 & 1 & -3 \end{pmatrix}$. Find PQ^T.

   Hence, find P^{-1}. **[7 marks]**

3. Solve the following inequalities
   a) $3x^2 - x > 3x + 4$. **[3 marks]**
   b) $|2x+1| < x+3$ **[4 marks]**

4. Find the values of x that satisfy the equation $2(9^x) = 7(3^x) - 3$. **[7 marks]**

5. a) The sum of the first six terms and the sixth term of an arithmetic sequence are S_6 and T_6 respectively. If $S_6 - T_6 = 35$, find the eleventh term of the sequence given that the first term is 3. **[5 marks]**

   b) Show that the $(r+1)^{th}$ term of binomial expansion $\left(x^2 - \frac{1}{x}\right)^{12}$ can be written as $T_{r+1} = (-1)^r \binom{12}{r} x^{24-3r}$. Hence find the term independent of x. **[6 marks]**

6. Given the matrix $A = \begin{pmatrix} 4 & -1 & 6 \\ y & 9 & 3 \\ x & 1 & 4 \end{pmatrix}$ and its cofactor matrix is $\begin{pmatrix} 33 & 2 & -17 \\ 10 & 4 & -6 \\ -57 & -6 & 37 \end{pmatrix}$.

   Show that $x = 2$ **[3 marks]**
   Hence, find

   a) the value of y **[3 marks]**
   b) the determinant, $|A|$. **[2 marks]**
   c) the adjoint matrix, $Adj(A)$. **[2 marks]**
   d) the inverse matrix, A^{-1}. **[2 marks]**

Final Answers

1. $z = \frac{16}{13} + \frac{24}{13}i$, $\frac{8}{\sqrt{13}}$, 0.983 rad

2. $3\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

   $P^{-1} = \frac{1}{3}\begin{pmatrix} -1 & -1 & 1 \\ -1 & -2 & 1 \\ 4 & 5 & -3 \end{pmatrix}$

3. a) $\left(-\infty, -\frac{2}{3}\right) \cup (2, \infty)$

   b) $\left(-\frac{4}{3}, 2\right)$

4. $x = 1$, -0.631

5. a) 23

   b) 495

6. $x = 2$

   a) $y = 1$

   b) 28

   c) $Adj(A) = \begin{pmatrix} 33 & 10 & -57 \\ 2 & 4 & -6 \\ -17 & -6 & 37 \end{pmatrix}$

   d) $A^{-1} = \frac{1}{28}\begin{pmatrix} 33 & 10 & -57 \\ 2 & 4 & -6 \\ -17 & -6 & 37 \end{pmatrix}$

Answer Scheme

1. $z = z_1 + \dfrac{1}{\bar{z}_2}$

$$z = 1 + 2i + \frac{1}{3 + 2i}$$

$$z = \frac{(1 + 2i)(3 + 2i) + 1}{3 + 2i}$$

$$z = \frac{3 + 2i + 6i - 4 + 1}{3 + 2i}$$

$$z = \frac{8i}{3 + 2i} \times \frac{3 - 2i}{3 - 2i}$$

$$z = \frac{24i + 16}{9 + 4}$$

$$z = \frac{16}{13} + \frac{24}{13}i$$

$$|z| = \sqrt{\left(\frac{16}{13}\right)^2 + \left(\frac{24}{13}\right)^2} = \frac{8}{\sqrt{13}}$$

$$\theta = \tan^{-1}\left(\frac{24}{16}\right) = 0.983\,\text{rad}$$

2.

$$Q^T = \begin{pmatrix} -1 & -1 & 1 \\ -1 & -2 & 1 \\ 4 & 5 & -3 \end{pmatrix}$$

PQ^T

$$= \begin{pmatrix} 3 & 6 & 3 \\ 3 & -3 & 0 \\ 9 & 3 & 3 \end{pmatrix} \begin{pmatrix} -1 & -1 & 1 \\ -1 & -2 & 1 \\ 4 & 5 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$= 3\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$PQ^T = 3I$$

$$P\left(\frac{1}{3}Q^T\right) = I$$

$$P^{-1} = \frac{1}{3}Q^T$$

$$= \frac{1}{3}\begin{pmatrix} -1 & -1 & 1 \\ -1 & -2 & 1 \\ 4 & 5 & -3 \end{pmatrix}$$

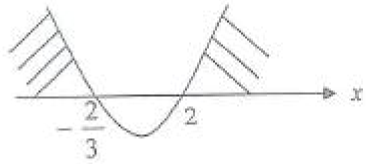
$$= \begin{pmatrix} \frac{-1}{3} & \frac{-1}{3} & \frac{1}{3} \\ \frac{-1}{3} & \frac{-2}{3} & \frac{1}{3} \\ \frac{4}{3} & \frac{5}{3} & -1 \end{pmatrix}$$

3.

$$3x^2 - x > 3x + 4$$

$$3x^2 - 4x - 4 > 0$$

$$(3x+2)(x-2) > 0$$



solution : $x < -\frac{2}{3}$ *or* $x > 2$

$$|2x+1| < x+3$$

$$-(x+3) < 2x+1 < x+3$$

$$-x-3 < 2x+1 \text{ and } 2x+1 < x+3$$

$$-3x < 4 \qquad\qquad x < 2$$

$$x > -\frac{4}{3}$$

Solution : $-\frac{4}{3} < x < 2$

4.

$$2(9^x) = 7(3^x) - 3$$
$$2(3^{2x}) = 7(3^x) - 3$$
Let $u = 3^x \rightarrow u^2 = 3^{2x}$
$$2u^2 = 7u - 3$$
$$2u^2 - 7u + 3 = 0$$
$$(2u - 1)(u - 3) = 0$$
$u = \frac{1}{2}$ or $u = 3$

$3^x = \frac{1}{2}$ $\quad 3^x = 3$

$x\log 3 = \log\frac{1}{2}$ $\quad x = 1$

$$x = \frac{\log\frac{1}{2}}{\log 3}$$
$$x = -0.631$$

5. a)

$$\frac{6}{2}\left[2(3) + 5d\right] - (3 + 5d) = 35$$
$$18 + 15d - 3 - 5d = 35$$
$$d = 2$$

$$T_{11} = 3 + (11 - 1)(2)$$
$$= 23$$

b) $\left(x^2 - \frac{1}{x}\right)^{12}$

$$T_{r+1} = \binom{12}{r}\left(x^2\right)^{12-r}\left(-\frac{1}{x}\right)^r$$
$$T_{r+1} = \binom{12}{r}(x)^{24-2r}(-1)^r\left(x^{-1}\right)^r$$
$$T_{r+1} = \binom{12}{r}(x)^{24-2r}(x)^{-r}(-1)^r$$

$$T_{r+1} = (-1)^r \binom{12}{r} (x)^{24-3r}$$

$$x^{24-3r} = x^0$$

$$24 - 3r = 0$$
$$r = 8$$
$$T_9 = (-1)^8 \binom{12}{8} = 23$$

6.

6	$\begin{vmatrix} 4 & 6 \\ x & 4 \end{vmatrix} = 4$ $16 - 6x = 4$ $x = 2$ or $-\begin{vmatrix} 4 & -1 \\ x & 1 \end{vmatrix} = -6$ $-(4 + x) = -6$ $4 + x = 6$ $x = 2$
(a)	$\begin{vmatrix} y & 9 \\ 2 & 1 \end{vmatrix} = -17$ $y = 1$
(b)	$\lvert A \rvert = 4\begin{vmatrix} 9 & 3 \\ 1 & 4 \end{vmatrix} + 1\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} + 6\begin{vmatrix} 1 & 9 \\ 2 & 1 \end{vmatrix}$ $= 132 + (-2) + (-102)$ $= 28$
(c)	$\text{Adj}(A) = \begin{pmatrix} 33 & 10 & -57 \\ 2 & 4 & -6 \\ -17 & -6 & 37 \end{pmatrix}$

(d)

$$A^{-1} = \frac{1}{28}\begin{pmatrix} 33 & 10 & -57 \\ 2 & 4 & -6 \\ -17 & -6 & 37 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{33}{28} & \frac{5}{14} & -\frac{57}{28} \\ \frac{1}{14} & \frac{1}{7} & -\frac{3}{14} \\ -\frac{17}{28} & -\frac{3}{14} & \frac{37}{28} \end{pmatrix}$$