1. Evaluate $\dfrac{1}{3-\sqrt{5}}-\dfrac{1}{1+\sqrt{5}}$. **[4 marks]**

2. Solve the inequality $x-1<x^2-3\leq 2x+5$. **[6 marks]**

3. If $\log_a\left(\dfrac{x}{a^2}\right)=3\log_a 2-\log_a(x-2a)$, express $x$ in terms of $a$. **[7 marks]**

4. a) Express $z=-\sqrt{3}-i$ in polar form.
**[3 marks]**

   b) Given that the complex number $z$ and its conjugate $\bar{z}$ satisfy the equation $z\bar{z}+2iz=12+6i$. Find the possible values of $z$.
**[4 marks]**

5. a) Given the first term of a geometric series is 40 and its sum to infinity is 60. Find the sum of the first forty terms, $S_{40}$ of the series.
**[5 marks]**

   b) Expand $\dfrac{1}{(1-2x)^3}$ in ascending powers of $x$ up to the term in $x^3$ and state the range of $x$ such that the expansion is valid. Hence, approximate $(0.9)^{-3}$.
**[8 marks]**

6. Given a matrix $T=\begin{pmatrix}1 & 1 & p\\ 0 & -q & 1\\ 3 & 2 & 1\end{pmatrix}$.

   a) Show that $|T|=1-q(1-3p)$. **[3 marks]**

   b) Find the values of $p$ and $q$ if $|T|=6$ and the minor for element $a_{22}$ is $-5$.
**[4 marks]**

   c) Hence, determine the inverse of $T$, $T^{-1}$ by using the adjoint method.
**[6 marks]**

**Final Answers**

1. 1

2. $\left[-2,-1\right)\cup\left(2,4\right]$

3. $x=4a$

4. a) $z=2\left[\cos\left(-\frac{5\pi}{6}\right)+i\sin\left(-\frac{5\pi}{6}\right)\right]$

   b) $z=3+3i$ , $z=3-i$

5. a) $S_{40}=60$

   b) $1+6x+24x^2+80x^3+\ldots$ ; $|x|<\frac{1}{2}$ ; 1.37

6. a) shown

   b) $p=2$ , $q=1$

   c) $\begin{pmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{5}{6} & -\frac{1}{6} \\ \frac{1}{2} & \frac{1}{6} & -\frac{1}{6} \end{pmatrix}$

**Answer Scheme**

1. $$\frac{1}{3-\sqrt{5}}\left(\frac{3+\sqrt{5}}{3+\sqrt{5}}\right)-\frac{1}{1+\sqrt{5}}\left(\frac{1-\sqrt{5}}{1-\sqrt{5}}\right)$$

$$=\frac{3+\sqrt{5}}{4}+\frac{(1-\sqrt{5})}{4}$$

$$=\frac{4}{4}$$

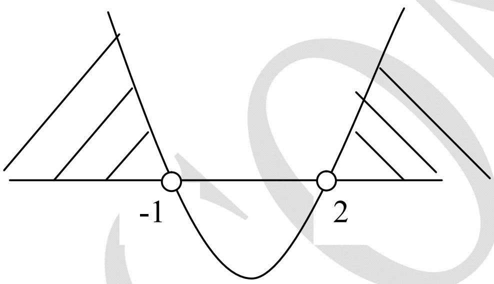
$$=1$$

2. $x-1<x^2-3\le 2x+5$

$x^2-3>x-1$

$x^2-x-2>0$

$(x+1)(x-2)>0$

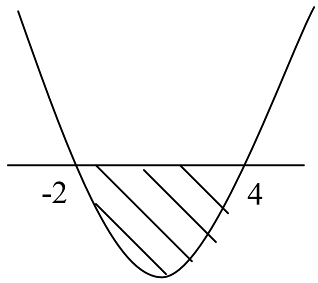



$(-\infty,-1)\cup(2,\infty)$

$x^2-3\le 2x+5$

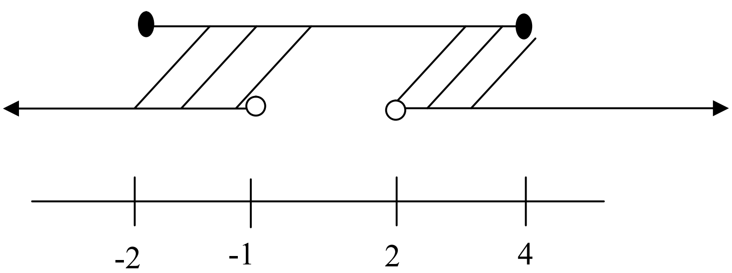
$x^2-2x-8\le 0$

$(x+2)(x-4)\le 0$




$[-2,4]$

Final solution :




$[-2,-1)\cup(2,4]$

3. $\log_a\left(\dfrac{x}{a^2}\right) = 3\log_a 2 - \log_a(x-2a)$

$$= \log_a 2^3 - \log_a(x-2a)$$

$$\log_a\left(\frac{x}{a^2}\right) = \log_a\left(\frac{8}{x-2a}\right)$$

$$\frac{x}{a^2} = \frac{8}{x-2a}$$

$$x^2 - 2ax = 8a^2$$

$$x^2 - 2ax - 8a^2 = 0$$

$$(x-4a)(x+2a) = 0$$

$x = 4a$ or $x = -2a$

Since $a > 0$, $x \neq -2a$

$\therefore \quad x = 4a$

4. a) $r = \sqrt{\left(-\sqrt{3}\right)^2 + (-1)^2} = 2$

$$\alpha = \tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{6}$$

$$\therefore \quad \arg, \theta = -\frac{5\pi}{6}$$

Polar form : $z = 2\left(\cos\left(-\dfrac{5\pi}{6}\right) + i\sin\left(-\dfrac{5\pi}{6}\right)\right)$

b) Let $z = x + yi$, $\bar{z} = x - yi$

$$(x+yi)(x-yi) + 2i(x+yi) = 12 + 6i$$

$$x^2 - y^2i^2 + 2xi + 2yi^2 = 12 + 6i$$

$$x^2 + y^2 - 2y + 2xi = 12 + 6i$$

$x^2 + y^2 - 2y = 12$ .............................. (1)

$2x = 6$ ...................................................(2)

$x = 3$, $\quad 9 + y^2 - 2y = 12$

$$y^2 - 2y - 3 = 0$$

$$(y-3)(y+1) = 0$$

$y = 3$ , $y = -1$

$\therefore \quad z = 3 + 3i$ , $z = 3 - i$

5. a) $S_\infty = \dfrac{a}{1-r}$, $a = 40$

$\dfrac{40}{1-r} = 60$

$\dfrac{40}{60} = 1 - r \quad \therefore \; r = \dfrac{1}{3}$

$S_n = \dfrac{a\left(1-r^n\right)}{1-r}$

$S_{40} = \dfrac{40\left[1-\left(\frac{1}{3}\right)^{40}\right]}{1-\frac{1}{3}} = 60$

b) $(1-2x)^{-3} = 1 + (-3)(-2x) + \dfrac{(-3)(-4)}{2!}(-2x)^2 + \dfrac{(-3)(-4)(-5)}{3!}(-2x)^3 + \ldots$

$= 1 + 6x + 24x^2 + 80x^3 + \ldots$

This expansion is valid for $|2x| < 1$ then $|x| < \dfrac{1}{2}$

$0.9 = 1 - 2x$
$\therefore \; x = 0.05$

$[1-2(0.05)]^{-3} = 1 + 6(0.05) + 24(0.05)^2 + 80(0.05)^3$
$= 1.37$

6. a) Expansion along the first row

$|T| = 1\begin{vmatrix} -q & 1 \\ 2 & 1 \end{vmatrix} - 1\begin{vmatrix} 0 & 1 \\ 3 & 1 \end{vmatrix} + p\begin{vmatrix} 0 & -q \\ 3 & 2 \end{vmatrix}$
$= (-q-2) - (0-3) + p(0+3q)$
$= -q - 2 + 3 + 3pq$
$= 1 - q(1-3p)$ shown

b) Given $|T| = 6$

$1 - q(1-3p) = 6$
$-q + 3pq = 5$ ...................... (1)

Minor, $a_{22} = -5$

$$\begin{vmatrix} 1 & p \\ 3 & 1 \end{vmatrix} = -5$$

$$1 - 3p = -5 \quad \Rightarrow \quad p = 2$$

Substitute $p = 2$ into (1) ;

$$-q + 3(2)q = 5 \quad \Rightarrow \quad q = 1$$

$$\therefore \ p = 2, \ \ q = 1$$

c) adj T $= C^T$

$$= \begin{pmatrix} +\begin{vmatrix} -1 & 1 \\ 2 & 1 \end{vmatrix} & -\begin{vmatrix} 0 & 1 \\ 3 & 1 \end{vmatrix} & +\begin{vmatrix} 0 & -1 \\ 3 & 2 \end{vmatrix} \\ -\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} & +\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix} \\ +\begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} & +\begin{vmatrix} 1 & 1 \\ 0 & -1 \end{vmatrix} \end{pmatrix}^T$$

$$= \begin{pmatrix} -3 & 3 & 3 \\ 3 & -5 & 1 \\ 3 & -1 & -1 \end{pmatrix}^T$$

$$= \begin{pmatrix} -3 & 3 & 3 \\ 3 & -5 & -1 \\ 3 & 1 & -1 \end{pmatrix}$$

$$T^{-1} = \frac{1}{|T|} \, adj \ T$$

$$= \frac{1}{6}\begin{pmatrix} -3 & 3 & 3 \\ 3 & -5 & -1 \\ 3 & 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{5}{6} & -\frac{1}{6} \\ \frac{1}{2} & \frac{1}{6} & -\frac{1}{6} \end{pmatrix}$$