LECTURE 1 OF 3

TOPIC : APPLICATIONS OF DIFFERENTIATION

SUBTOPIC : 10.1 Extremum Problems

OBJECTIVES : At the end of the lesson students will be able to :

1. Find the critical points.
2. Find the relative extremum using the First Derivative Test.
3. Find the relative extremum using the Second Derivative Test
4. Solve optimization problems.

SET INDUCTION:

Ask students to observe the movement of the coaster and then identify its maximum or minimum position.

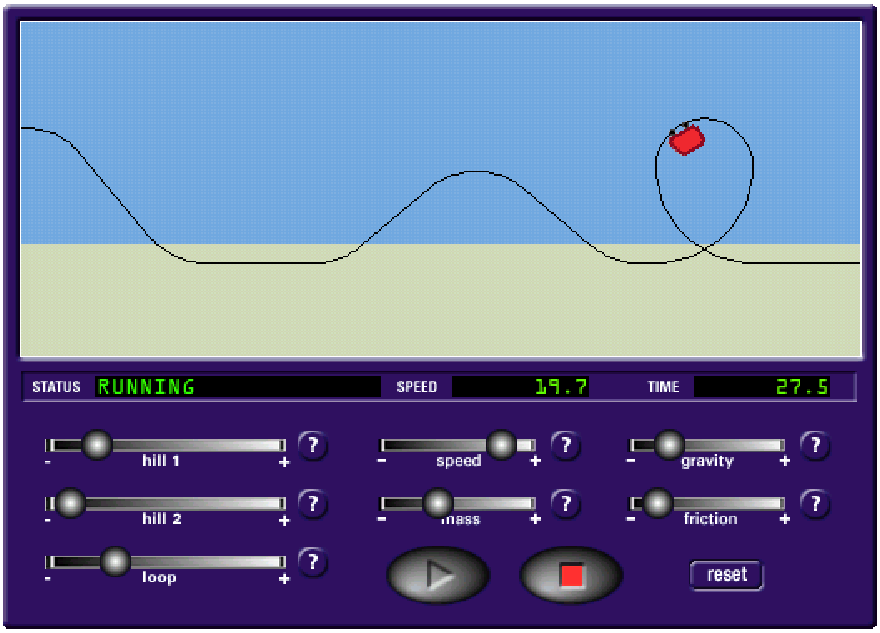


Figure 2: Application of extremum values

From the simulation above, student could predict the maximum value and minimum value.
As a conclusion, the lecturer verify the answer and show them a few more different situations.

10.1 EXTREMUM PROBLEMS

1. **A critical point** for a function $f(x)$ is any value of x in the domain of f at which $f'(x) = 0$ or $f'(x)$ is not defined.

2. The critical point where $f'(x) = 0$ are called **stationary point** and the value of the function at that point a **stationary value**.

Example 1

Find the stationary points and stationary values of the function $4x^3 + 15x^2 - 18x + 7$.

Solution

Example 2

Find the critical numbers of $f(x)=x^{\frac{3}{5}}(4-x)$.

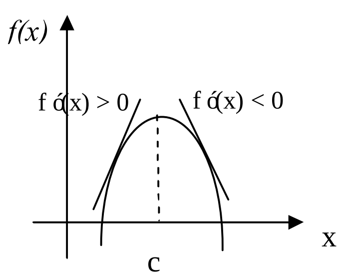
Solution

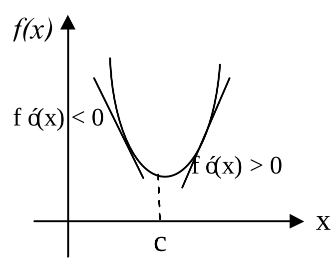
The First Derivative Test

Suppose that c is a critical number of a function f that is continuous on (a,b).

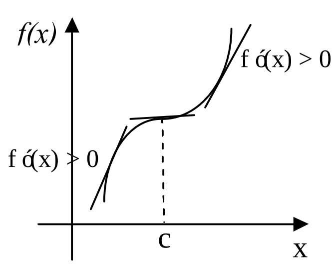
a. If $f'(x) > 0$ for (a,c) and $f'(x) < 0$ for (c,b), then f has a **local maximum** at c.
b. If $f'(x) < 0$ for (a,c) and $f'(x) > 0$ for (c,b), then f has a **local minimum** at c.
c. If $f'(x)$ does not change sign at c, then f has **no local extremem** at c.

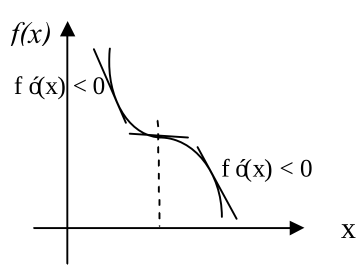
It is easy to remember the First Derivative Test by visualizing diagrams such as those in Figure 3.22





a. Local maximum b. Local minimum





c. No extremum d. No extremum

Example 3

Find the maximum or minimum values of the function $y = 4x^3 + 15x^2 - 18x + 7$.

Solution

Example 4

Find the stationary points of the curve $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ and determine the nature of that points.

Solution

Example 5

Find the relative extremum of $f(x)=\frac{9}{5}x^{\frac{5}{3}}-18x^{\frac{2}{3}}$.

Solution

LECTURE 2 OF 3

TOPIC : APPLICATIONS OF DIFFERENTIATION

SUBTOPIC : 10.1 Extremum Problem

OBJECTIVES : At the end of the lesson students will be able to

1. Find the relative extremum using the Second Derivative Test.
2. Solve the Optimum Problems

CONTENTS

The Second Derivative Test

Another application of the second derivative is in finding maximum and minimum values of a function.

Suppose $f''(x)$ is continuous on an open interval that contains c

a. If $f'(x)=0$ and $f''(x)>0$, then f has a local minimum at c

b. If $f'(x)=0$ and $f''(x)<0$, then f has a local maximum at c

c. If $f'(x)=0$ and $f''(x)=0$, the test fails, it is inconclusive. Thus we must use the first derivative test.

Example 1

By using the Second Derivative Test, find the relative extremum of $f(x)=x^3-3x^2-24x+32$.

Solution

Example 2

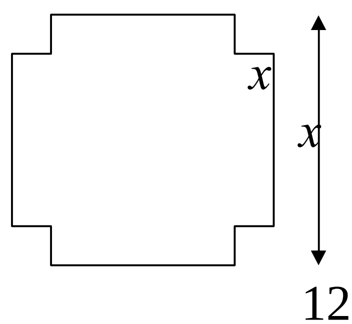
By using the Second derivative Test, find the relative extremum of the function
$f(x) = x^4 - 4x^3$.

Solution

Optimum Problems

Example 1

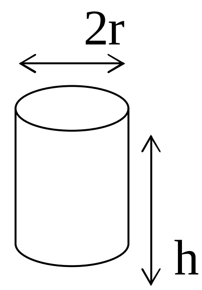
An open-top box is to be made by cutting small congruent squares from the corners of a 12 cm by 12 cm sheet of tin and bending up the sides. How large should the squares cut from the corners be to make the box hold as much as possible?



Solution

Example 2

You have been asked to design a 1 liter oil can shaped like a right circular cylinder.

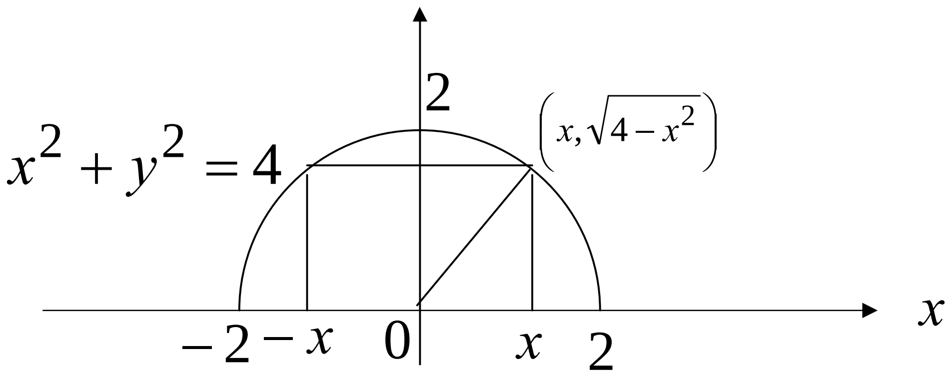


Which dimensions will use the least material?

Solution

Example 3

A rectangle is to be inscribed in a semi circle of radius 2. What is the largest area the rectangle can have, and what are its dimensions?



Solution

Example 4

A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 800 m of wire at your disposal what is the largest area you can enclose, and what are its dimensions?

Solution

LECTURE 3 OF 3

TOPIC : APPLICATIONS OF DIFFERENTIATION

SUBTOPIC : 10.2 Rate of Change

OBJECTIVES : At the end of the lesson students will be able to :

1. Solve problem regarding rate of change including related rates.

SET INDUCTION:

Lecturer explains :

The rate of change in velocity (the speed of an object in a certain direction) is known as acceleration. Whether an object is speeding up, slowing down, or changing direction, it is accelerating. Most amusement park rides involve acceleration. On a downhill slope or a sharp curve, a ride will probably increase in velocity or accelerate. While moving uphill or in a straight line, it may decrease in velocity or decelerate. When the force of gravity pulling a roller coaster down hill causes the roller coaster to go faster and faster, it is accelerating. When the force of gravity causes a roller coaster to go slower and slower when it climbs a hill, the roller coaster is decelerating or going slower. The acceleration of a roller coaster depends on its mass and how strong is the force that is pushing or pulling it.

At the same time, lecturer plays back the simulation as shown below.

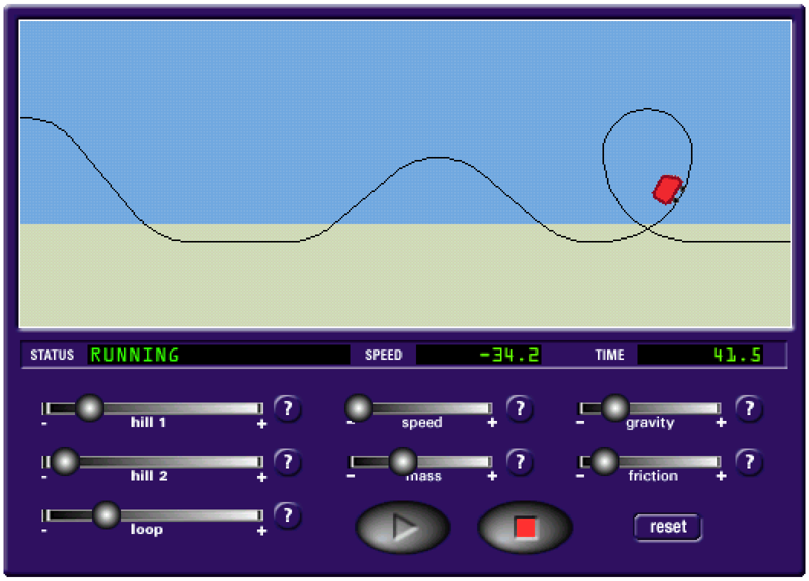


Figure 4 : Rate of change.

CONTENTS

- If $y = f(x)$, then $\dfrac{dy}{dx}$ is the rate of change of y with respect to x.
- They are often, but not always, rates of change with respect to time.

- The rate of change of quality Q, $\left(\frac{dQ}{dt}\right)$ means:

  the rate of change of Q with respect to time t.

- Rate of ñ***Increase***ò is <u>positive</u>.
- Rate of ñ***Decrease***ò is <u>negative</u>.
- Rate of change of y and rate of change of x can be shown as: $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

Strategies for solving related rate problems

1. Draw a picture and name the variables and constants. Use t for time and assume all variables are differentiation function of t.
2. Write down the numerical information (in term of the symbols you have chosen)
3. Write down what you are asked to find (usually a rate, expressed as a derivative)
4. Write an equation that relates the variables you may have to combine two or more equations to get a single equation that relates the variable whose rate you want to the variable whose rate you know.
5. Differentiation with respect to t to express the rate you want in terms of the rate and variables whose values you know.
6. Evaluate/calculate.

Example 1

Gas is escaping from a spherical balloon at the rate of $2\ \text{m}^3/\text{min}$. How fast is the surface area shrinking when the radius is 12 cm?

Solution

Example 2

A container in the shape of a hollow cone of semi-vertical angle 30° is held with its vertex pointing downwards. Water is poured into the cone at the rate of $5\text{cm}^3\text{s}^{-1}$. Find the rate at which the depth of water in the cone is increasing when this depth is 10 cm.

Solution

Example 3

The total cost of manufacturing p boxes of chocolates (a function of t) is given by $C(p) = 3p^2 + 2p + 300$ where $p(t) = 2t^2 + 75t$.

Compute the rate of change of the total cost with respect to time when $t = 1$.

Solution

Example 4

A 8 meter ladder rest against a vertical wall (*see figure below*). If the bottom of the ladder is sliding away from the base of the wall at the rate of $0.4\,\text{m/sec}$, how fast is the top of the ladder moving down the wall when the bottom of the ladder is 2 meter from the base?

Solution