LECTURE 1 OF 8
TOPIC: 9.0 DIFFERENTIATION
SUBTOPIC: 9.1 Derivative of a Function

OBJECTIVE:
At the end of the lesson students are able to:

(a) determine the derivative of a function *f(x)* using the first principle

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

9.1 Derivative of a function

1. If $y = f(x)$, then $\dfrac{dy}{dx} = f'(x) = \lim_{h \to 0}\left[\dfrac{f(x+h) - f(x)}{h}\right]$. This is called the first principle.

   $f'(x)$ is the gradient of the tangent to the curve, $y = f(x)$.

2. The process of finding $f'(x)$ is called differentiation.

3. The notation that is sometimes used is that if $y = f(x)$ is

   $\dfrac{dy}{dx}$ or y' or $\dfrac{d}{dx}f(x)$ or $f'(x)$

   For example, the derivative for $y = x^3$ can be written as $\dfrac{d}{dx}\left(x^3\right)$ or just $\dfrac{dy}{dx}$.

Example 1
By using first principles, find $f'(x)$ for each the following.

(a) $f(x) = ax^2 + bx + c$ (c) $f(x) = \sqrt{x}$

(b) $f(x) = x^2$ (d) $f(x) = \dfrac{1}{3x+5}$

LECTURE 2 OF 8
TOPIC: 9.0 DIFFERENTIATION
SUBTOPIC: 9.1 Derivative of a function
9.2 Rules of Differentiation

OBJECTIVE:
At the end of the lesson students are able to
9.1 (b) determine the differentiability of a function at $x = a$

9.2 (a) apply the rules of differentiation:
i. Basic rule
ii. Sum rule

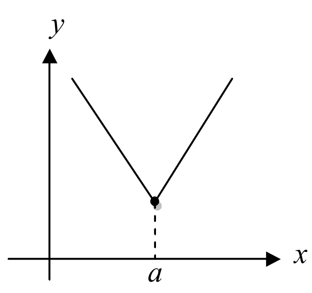
9.1 b) To Determine the Differentiability of a Function at $x = a$

A function f is differentiable at a point at $x = a$ if the derivative $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ exist at $x = a$.

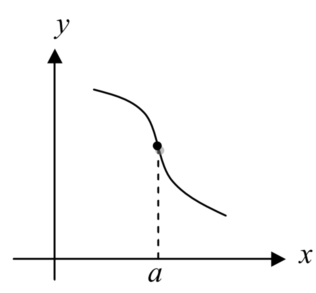
$$f'(a) = \lim_{x \to a^-} \frac{f(x) - f(a)}{x - a} = \lim_{x \to a^+} \frac{f(x) - f(a)}{x - a}$$

If a function is differentiable at a point, then the function is also continuous at that point. However, a function may be continous at a point but not differentiable. This happens at point where there are corners or vertical tangents.

For example,



Corner



Vertical tangent

The slopes have different limits from the left and from the right, hence f is not differentiable at $x = a$.

The Derivatives at a Point

The derivative of the function *f* at $x = a$ is

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Example 2

By using first principle, find $f^{'}(a)$ for each of the following.

(a) $f(x) = x^2 + 3$, $\boldsymbol{a} = 3$

(b) $f(x) = \dfrac{1}{\sqrt{x+1}}$, $a = 8$

Example 3

Given $f(x) = \begin{cases} 2x & ,x < 0 \\ x^2 & ,x \geq 0 \end{cases}$.

Is *f* differentiable at $x = 0$?

Example 4

Determine whether the function $f(x) = |x|$ is differentiable at x=0.

Example 5

Show that $f(x) = \begin{cases} x^2 + 3 & ,x \leq 1 \\ x + 3 & ,x > 1 \end{cases}$ is continous but not differentiable at $x = 1$.

9.2 Rules of Differentiation

a) Basic Rule

i) if $f(x) = k$, then $f'(x) = 0$

ii) if $f(x) = x^n$, then $f'(x) = n x^{n-1}$

iii) if $f(x) = kx^n$, then $f'(x) = knx^{n-1}$

(i) The Derivative of a Constant Function, $f(x) = k$

If k is any real number, the derivative of $f(x) = k$ is $f'(x) = 0$.

In other words, the derivative of a constant function is zero.

Example 6

Find the derivatives of

(a) $f(x) = 8$ (b) $f(x) = 0.73$ (c) $f(x) = \sqrt{3}$

(ii) The Derivative of a Function of the Form $f(x) = x^n$

If n is a real number, the derivative of $f(x) = x^n$ is $f'(x) = nx^{n-1}$

Example 7

(a) $\frac{d}{dx}\left(x^4\right)$

(b) $\frac{d}{dx}\left(x^{\frac{3}{2}}\right)$

(c) $\frac{d}{dx}\left(\frac{1}{x^2}\right)$

(d) $\frac{d}{dx}\left(\sqrt{x}\right)$

(iii) The Derivative of a Function of the Form $f(x) = kx^n$

If k and n is a real number, the derivative of $f(x) = kx^n$ is $f'(x) = knx^{n-1}$

Example 8

Find the derivatives of

(a) $f(x) = 5x$

(b) $f(x) = -x^{12}$

(c) $h(x) = 3\pi x^3$

(d) $g(x) = \frac{5x^4}{3}$

(e) $m(x) = \frac{12}{\sqrt{x}}$

b) The Sum Rule of Differentiation

If $u(x)$ and $v(x)$ are differentiable functions,

the derivative of $f(x) = u(x) + v(x)$ is $f'(x) = u'(x) + v'(x)$

and the derivative of $f(x) = u(x) - v(x)$ is $f'(x) = u'(x) - v'(x)$

Example 9

(a) $\frac{d}{dx}\left(x^4 + x^2\right)$

(b) $\frac{d}{dx}\left(2x^2 - 5\sqrt{x}\right)$

(c) $\frac{d}{dx}\left(3\sqrt{x} + \frac{2}{\sqrt{x}} + \sqrt{2}\right)$

Note : The differentiation of a function should be with respect to the independent variable for example, if

i) $y = f(x)$ then $\frac{dy}{dx} = f'(x)$

ii) $y = h(t)$ then $\frac{dy}{dt} = h'(t)$

and so on.

LECTURE 3 OF 8
TOPIC: 9.0 DIFFERENTIATION
SUBTOPIC: 9.2 Rules of Differentiation

OBJECTIVE:
At the end of the lesson students are able to

(a) Apply the rules of differentiation:

iii. Product rule

iv. Quotient rule

v. Chain rule

c) The Product Rule of Differentiation

If $u(x)$ and $v(x)$ are differentiable functions, the derivative of

$$f(x) = u(x)v(x)$$

is
$$f'(x) = u(x)v'(x) + v(x)u'(x)$$

The Product Rule can be written as $\frac{d}{dx}(uv) = uv' + vu'$

Example 10

Differentiate each of the following functions

(a) $p(x) = (3x^2 - 1)(7 + 2x^3)$

(b) $h(x) = 3x^2(5x + 1)$

(c) $f(x) = \left(\frac{2}{x} + 1\right)\left(\frac{1}{x^2} - 3\right)$

d) The Quotient Rule of Differentiation

If $u(x)$ and $v(x)$ are differentiable functions, the derivative of

$f(x)=\dfrac{u(x)}{v(x)}$ is $$f'(x)=\frac{v(x)u'(x)-u(x)v'(x)}{[v(x)]^2}$$

The Quotient Rule can be written as $\dfrac{d}{dx}\left(\dfrac{u}{v}\right)=\dfrac{vu'-uv'}{v^2}$

Example 11

Differentiate each of the following functions,

(a) $f(x)=\dfrac{x^2-1}{x^4+1}$

(b) $f(x)=\dfrac{3-\left(\dfrac{1}{x}\right)}{x+5}$

Example 12

a) $\dfrac{d}{dx}\left[\dfrac{(1-2x)(3x+2)}{5x-4}\right]$

b) $\dfrac{d}{dx}\left[x\left(1-\dfrac{2}{x+1}\right)\right]$

e) The Chain Rule of Differentiation

If $y=f(u)$ is a differentiable function of u and

$u=g(x)$ is a differentiable function of x, then

$y=f[g(x)]$ is a differentiable function of x and

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$$

Example 13

Find $\frac{dy}{dx}$ if $y = u^3 - 3u^2 + 1$ and $u = x^2 + 2$

Example 14

Differentiate the following with respect to *x*, using the Chain Rule.

(a) $y = (x+4)^3$ (c) $y = \sqrt{7x-2}$

(b) $y = \frac{1}{4x+9}$

The General Power Rule of Differentiation

In general, any composite function of the form $y = [f(x)]^n$, involving some function $f(x)$ raised to a rational power *n*, is called the **General Power Rule**, and it is a special case of the Chain Rule.

Let $y = u^n$, where $u = f(x)$.

Then $\frac{dy}{du} = nu^{n-1}$, and $\frac{du}{dx} = f'(x)$

Using the Chain Rule,

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

So $$\frac{dy}{dx} = [nu^{n-1}][f'(x)]$$

$$\boxed{\frac{d}{dx}[f(x)]^n = [n[f(x)]^{n-1}][f'(x)]}$$

Example 15

Find the derivatives of

(a) $f(x) = (x^5+1)^3$ (b) $y = (2x^4 - 9x + 6)^4$

Example 16

Differentiate $g(x) = \sqrt{\frac{x}{1-x}}$

Higher Order Derivatives

Occasionally, it is useful to differentiate the derivative of a function. In this context, we shall refer to f' as the first derivative of *f* and to the derivative of f' as the second derivative of *f*.

We could denote the second derivative by $(f')'$, but for simplicity we write f''. Other higher-order derivatives are defined and denoted by f'''. In general, for $n > 3$, the n^{th} derivative of *f* is denoted by $f^{(n)}$, for example, $f^{(4)}$ or $f^{(5)}$.

In *Leibniz* notation, higher derivatives for $y = f(x)$ are denoted as follows

First derivative	y'	$f'(x)$	$\frac{dy}{dx}$	or	$\frac{d}{dx}[f(x)]$
Second derivative	y''	$f''(x)$	$\frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d^2y}{dx^2}$	or	$\frac{d^2}{dx^2}[f(x)]$
Third derivative	y'''	$f'''(x)$	$\frac{d}{dx}\left[\frac{d^2y}{dx^2}\right] = \frac{d^3y}{dx^3}$	or	$\frac{d^3}{dx^3}[f(x)]$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	or	$\vdots$
n[th]	$y^{(n)}$	$f^{(n)}(x)$	$\frac{d^ny}{dx^n}$	or	$\frac{d^n}{dx^n}[f(x)]$

Example 17

Find the first, second and third order derivatives of

$$p(x) = -2x^4 + 9x^3 - 5x^2 + 7$$

LECTURE 4 OF 8

TOPIC : 9.0 DIFFERENTIATION

SUBTOPIC : 9.3 Differentiation of Exponential, Logarithmic and Trigonometric Functions

OBJECTIVES : At the end of the lesson students are able to

(a) Determine the derivatives of the functions:
i. $a^x, a^{f(x)}, e^x, e^{f(x)}$
ii. $\ln x$ and $\ln f(x)$

SET INDUCTION:

Before introducing the derivative of exponential and logarithmic functions, recall the derivative of functions which students have learn in previous lectures.

$$\frac{d}{dx}[k] = 0$$
$$\frac{d}{dx}[kx + c] = k$$
$$\frac{d}{dx}[kx^n] = nkx^{n-1}$$
$$\frac{d}{dx}[(f(x))^n] = n[f(x)]^{n-1}$$

9.3 (a) (i) Derivatives of Exponential Functions

1. For a simplest index function a^x,

$$\frac{d}{dx}(a^x) = a^x \ln a$$

Proof:

$$y = a^x$$

taking ln of both sides,

$$\ln y = \ln a^x$$
$$\ln y = x \ln a$$

Differentiating both sides with respect to x

$$\frac{1}{y}\frac{dy}{dx} = x(0) + (\ln a)(1)$$
$$= \ln a$$

$$\frac{dy}{dx} = y \ln a$$
$$= a^x \ln a$$

If m is a scalar,

$$\frac{d}{dx}\left(a^{mx}\right) = ma^{mx} \ln a$$

2. For the general exponential function, $a^{f(x)}$ where $f(x)$ is a function of x, the derivative is given by

$$\frac{d}{dx}\left(a^{f(x)}\right) = f'(x)a^{f(x)} \ln a$$

Example 18

Differentiate the following with respect to x

(a) $y = 5^x$ (b) $y = 4^{3x+5}$ (c) $y = 3^{5-2x}$ (d) $y = 4^{-x+1}$

3. For a simplest exponential function e^x,

$$\frac{d}{dx}\left(e^x\right) = e^x$$

If m is a scalar,

$$\frac{d}{dx}\left(e^{mx}\right) = me^{mx}$$

4. For the general exponential function, $e^{f(x)}$ where $f(x)$ is a linear function of x, the derivative is given by

$$\frac{d}{dx}\left(e^{f(x)}\right) = f'(x)e^{f(x)}$$

Proof (by using chain rule)

$$y = e^{f(x)}$$

Let $u = f(x)$ $\Rightarrow$ $y = e^u$

$$\frac{du}{dx} = f'(x) \qquad \frac{dy}{du} = e^u$$

So, $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

$$\frac{dy}{dx} = e^u \times f'(x)$$
$$= f'(x)e^{f(x)}$$

Example 19:

Find $\frac{dy}{dx}$ of the following:

a) $y = e^{\frac{1}{2}x}$ b) $y = e^{2x-1}$ c) $y = 7e^{-3x+5}$

Example 20

Find the derivatives of the following functions

(a) $y = xe^{x}$ (b) $y = \frac{3x+2}{e^{x}}$

(c) $y = \frac{3}{x}\, e^{-2x}$ (d) $y = \frac{e^{x}}{e^{x}+e^{-x}}$

9.3 (a) (ii) Derivative of Logarithmic Functions

1. The natural logarithmic function $\ln x$ is differentiable for all $x > 0$.

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

Proof:

$$y = \ln x$$
$$e^{y} = x$$
$$e^{y}\frac{dy}{dx} = 1$$
$$\frac{dy}{dx} = \frac{1}{e^{y}}$$
$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

Natural logarithmic functions of the form $y = \ln(f(x))$, where $f(x)$ is linear function of x, $\frac{d}{dx}[\ln f(x)] = \frac{f'(x)}{f(x)}$

Proof: (by using the Chain Rule)

$$y = \ln(f(x))$$

Let $u = f(x) \Rightarrow y = \ln u$

$\frac{dy}{du} = \frac{1}{u}$ and $\frac{du}{dx} = f'(x)$.

Using the Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$
$$= \left(\frac{1}{u}\right)(f'(x))$$
$$= \frac{f'(x)}{f(x)}$$

Example 21

Differentiate the following with respect to x.

(a) $\ln 3x$ (b) $\frac{1}{2}\ln(5-3x)$ (c) $y = 7\ln(-2x+1)$

Example 22

Find the derivatives of the following

(a) $y = x\ln x$ (b) $y = \frac{\ln x}{e^x}$

LECTURE 5 OF 8

TOPIC : 9.0 DIFFERENTIATION

SUBTOPIC : 9.3 Differentiation of Exponential, Logarithmic and Trigonometric Functions

OBJECTIVES : At the end of the lesson students are able to

(b) Determine the derivatives of the functions:

iii. $\sin x, \cos x, \tan x, \sec x, \cos ecx, \cot x$

iv. $\sin u, \cos u, \tan u, \sec u, \cos ecu, \cot u$

v. $\sin^n x, \cos^n x, \tan^n x, \sec^n x, \cos ec^n x, \cot^n x$

Derivatives of the functions sin *x*, cos *x*, tan x, sec *x*, cot *x*, and cosec *x*

a) $\frac{d}{dx}(\sin x) = \cos x$	d) $\frac{d}{dx}(\cot x) = -\cos ec^2 x$
b) $\frac{d}{dx}(\cos x) = -\sin x$	e) $\frac{d}{dx}(\sec x) = \sec x \tan x$
c) $\frac{d}{dx}(\tan x) = \sec^2 x$	f) $\frac{d}{dx}(\cos ecx) = -\cos ec\, x \cot x$

Note **:** Differentiation of the above formulae are only true for angles which are measured in radians.

Example 23

Differentiate with respect to *x*.

a) 2 sin *x* ï 3 cos *x* b) 4 tan *x* - $\frac{5}{x}$ c) 4 cos *x* + 3 tan *x* d) 4 sin *x* ï 5 ï 6 cos *x*

Derivatives of the functions sin *u*, cos *u*, tan *u*, sec *u*, cot *u*, and cosec *u*

a) $\frac{d}{dx}(\sin u) = \cos u \frac{d}{dx}(u)$

b) $\frac{d}{dx}(\cos u) = -\sin u \frac{d}{dx}(u)$

c) $\frac{d}{dx}(\tan u) = \sec^2 u \frac{d}{dx}(u)$

d) $\frac{d}{dx}(\cot u) = -\operatorname{cosec}^2 u \frac{d}{dx}(u)$

e) $\frac{d}{dx}(\sec u) = \sec u \tan u \frac{d}{dx}(u)$

f) $\frac{d}{dx}(\operatorname{cosec} u) = -\operatorname{cosec} u \cot u \frac{d}{dx}(u)$

Example 24
Find the derivatives of :

a) cos ($5x + 4$) b) sin ($\ln x$) c) tan ($\frac{1}{6}\pi - 3x$) d) sec ($2x + 3$)

Derivatives of the functions $\sin^n x, \cos^n x, \tan^n x, \sec^n x, \operatorname{cosec}^n x, \cot^n x$

a) $\frac{d}{dx}(\sin^n u) = n\sin^{n-1} u \cos u \frac{d}{dx}(u)$

b) $\frac{d}{dx}(\cos^n u) = -n\cos^{n-1} u \sin u \frac{d}{dx}(u)$

c) $\frac{d}{dx}(\tan^n u) = n\tan^{n-1} u \sec^2 u \frac{d}{dx}(u)$

d) $\frac{d}{dx}(\cot^n u) = -n\cot^{n-1} u \operatorname{cosec}^2 u \frac{d}{dx}(u)$

e) $\frac{d}{dx}(\sec^n u) = n\sec^{n-1} u \sec u \tan u \frac{d}{dx}(u)$

f) $\frac{d}{dx}(\operatorname{cosec}^n u) = -n\operatorname{cosec}^{n-1} u \operatorname{cosec} u \cot u \frac{d}{dx}(u)$

Example 25
Differentiate each of the following functions with respect to *x*.
a) $y = \sec^2 x$ b) $y = \sin^3 (x^2 + 1)$ c) $y = \operatorname{cosec}^4 x$ d) $y = \cot^2 3x$

LECTURE 6 OF 8

TOPIC : 9.0 DIFFERENTIATION

SUBTOPIC : 9.3 Differentiation of Exponential, Logarithmic and Trigonometric Functions

OBJECTIVE:
At the end of the lesson students are able to
(b) solve problems involving the combination of differentiation rules

Solve Problems of Differentiation

Example 26
Find the first derivatives of the functions given below

(a) $f(x) = e^{3x}\left(x^2 - 1\right)$

(b) $f(x) = \dfrac{e^{2x}}{1+e^x}$

(c) $f(x) = x^2 \ln(3x+1)$

(d) $f(x) = x \sin\dfrac{1}{x}$

Example 27
Given $y = (3+4x)\,e^{-2x}$, find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ and show that $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 4y = 0$

Example 28
Find the derivatives of the function given below

(a) $f(x) = x^2 e^{-3x+2}$

(b) $f(x) = 2^{x^3 \ln x}$

(c) $h(x) = x^2 \ln x$

(d) $y = \sqrt{x^2 + \sec(2x+3)}$

(e) $y = \sin x \cos^3 x$

LECTURE 7 OF 8

TOPIC: 9.0 DIFFERENTIATION

SUBTOPIC: 9.4 Implicit Differentiations

OBJECTIVES:

At the end of the lesson students are able to

(a) find the first and second derivatives implicitly.

Implicit Differentiation

The function $y = f(x)$ is said to be in explicit form. For example, the functions $y = 2x+1$, $y = \sqrt{x^2 - 25}$ and $y = \left(\frac{x-1}{x+3}\right)^2$ are all functions defined explicitly.

But not all functions can easily be transposed to the form $y = f(x)$. For example, the functions $y + 2xy - y^2 = 3$. It cannot be written in the form $y = f(x)$, and $y = f(x)$ is implied by the functions $y + 2xy - y^2 = 3$. In other words, function $y + 2xy - y^2 = 3$ is an implicit function.

Suppose $y^2 + 4x = 6x^2$

$$\frac{d}{dx}\left(y^2\right) + \frac{d}{dx}(4x) = \frac{d}{dx}\left(6x^2\right)$$

Thus $2y\frac{dy}{dx} + 4 = 12x$

The First Order Derivatives

Example 29

Find $\frac{dy}{dx}$ in term of x and y if

(a) $y^3 + 6x = x^2$ (b) $3y^2 + 2y + xy = x^3$ (c) $x\ln y = e^{2x}$

The Second Order Derivatives

Example 30

Given that, equation of a curve is $3x^2 + 4y^2 = 15xy - 6$. Find the value of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at the point (1, 3).

Example 31

Given that $xy = \sin 3x$, show that $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 9xy = 0$.

Example 32

The curve $x^3 + y^3 = kx + 2y$ passes through the point (1, -1).

(a) Determine the value of the constant *k*.

(b) Find the value of $\frac{dy}{dx}$ at the point (1, -1).

(c) Show that $(3y^2 - 2)\frac{d^2y}{dx^2} + 6y\left(\frac{dy}{dx}\right)^2 + 6x = 0$.

LECTURE 8 OF 8

TOPIC : 9.0 DIFFERENTIATION

SUBTOPIC : 9.5 Parametric Differentiations

OBJECTIVES:

At the end of the lesson, student will be able to:

(a) Use parametric differentiation to find the first and second derivatives

First derivative of parametric differentiation

Definition :

If x and y are both function of the same independent variable t, then the equation

$$x = f(t) \quad \text{and} \quad y = g(t)$$

are called parametric equations and t is called the parameter.

We can differentiate parametric equation using the chain rule

$$\boxed{\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}}$$

Example 33

Find $\frac{dy}{dx}$ in terms of the parameter t if

(a) $x = 2t^3,\ y = 4t^2 + 1$ (b) $x = \frac{3}{t},\ y = \sqrt{1+t^2}$

Example 34

If $x = \frac{t}{1+t}$ and $y = \frac{t^2}{1+t}$, $t \neq -1$, show that $\frac{dy}{dx} = t(t+2)$.

Example 35

Find the value of $\frac{dy}{dx}$ if $x = t - \frac{1}{t},\ y = 2t + \frac{4}{t}$ when $t = 2$.

Example 36

Find $\frac{dy}{dx}$ in terms of the parameter t if $x = -\cos 3t$ and $y = 2\sin^2 3t$

Second derivatives of parametric equations

The answer for $\frac{dy}{dx}$ are usually in terms of the parameter, t, so in order to find the second derivative $\frac{d^2y}{dx^2}$, we will apply the chain rule again.

If $x = f(t)$ and $y = g(t)$, (t is parameter)

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right) \cdot \frac{dt}{dx}$$

Thus
$$\boxed{\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\left(\frac{dx}{dt}\right)}}$$

Example 37

The terms of parameter t is given by

$x = \frac{2}{(1-t)^2}$ and $y = \frac{t}{1-t}$

(a) Show that $\frac{dy}{dx} = \frac{1}{4}(1-t)$ (b) Find $\frac{d^2y}{dx^2}$

Example 38

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of t given that $x = \frac{1}{t}$, $y = 3t^2 + 2$.

Example 39

If $x = \frac{1+t}{1-t}$ and $y = (1+t)(1-t^2)$ where t is a parameter, find

(a) $\frac{dy}{dx}$ (b) $\frac{d^2y}{dx^2}$

Example 40

If $x = te^{2t}$ and $y = e^{2t}$, find

(a) $\frac{dy}{dx}$ (b) $\frac{d^2y}{dx^2}$

when $t = 0$.