LECTURE 1

TOPIC : 4.0 MATRICES AND SYSTEMS OF LINEAR EQUATIONS

SUBTOPIC:4.1 Matrices

LEARNING OUTCOMES:

At the end of the lesson, students should be able to:

(a)Perform operations on matrices such as addition, subtraction,scalar multiplication and multiplication of two matrices.

SET INDUCTION

The result of Euro 2006 (Group B)

Team	GP	W	D	L	Pts
France	3	2	1	0	7
England	3	2	0	1	6
Croatia	3	0	2	1	2
Switzerland	3	0	1	2	2

The above standing shows **MATRIX**form.

CONTENT

Definition

A matrix is a rectangular array of real numbers that are arranged in rows and columns.

Example: $\begin{bmatrix} 4 & 7 & 3 \\ 2 & 5 & 9 \end{bmatrix}$

This matrix which has 2 rows and 3 column is called 2x3 matrix and we say that having order 2 by 3. Each number in matrix is called an entry or element for example $a_{13} = 3$, $a_{23} = 9$, $a_{22} = 5$

Types of Matrices

- ***Row Matrix*** is a (1 x n) matrix (one row);

  $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \end{bmatrix}$

  Example 1:

  $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \end{bmatrix}$;
  $B = \begin{bmatrix} 1 & 0 & 7 & 8 & 4 & 3 & 5 \end{bmatrix}$

- ***Column Matrix*** is a (m x 1) matrix (one column);

$$A = \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ \vdots \\ a_{m1} \end{bmatrix}$$

Example2:

$$A = \begin{bmatrix} 0 \\ -4 \end{bmatrix}, \qquad B = \begin{bmatrix} 2 \\ 3 \\ 5 \\ 7 \end{bmatrix}$$

- ***Zero Matrix*** is a (m x n) matrix which every entry is zero, and denoted by O.

Example 3:

$$o = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \; O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

- ***Diagonal Matrix***

Let $$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1m} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2m} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3m} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mm} \end{bmatrix}$$

- The diagonal entries of A are $a_{11}, a_{22}, a_{33}, ..., a_{mm}$
- A square matrix which non-diagonal entries are all zero is called a diagonal matrix.

Example4:

(a) $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ (b) $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

(c) $C = \begin{bmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & b \end{bmatrix}$

- ***Identity Matrix*** is a diagonal matrix in which all its diagonal entries are 1, and denoted by I.

Example5:

a) $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_{2\times 2}$

b) $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_{3\times 3}$

- ***Lower Triangular Matrix*** is a square matrix and $a_{ij} = 0$ for $i < j$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Example 6:

$$\text{A} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 2 & 0 \\ 3 & 2 & 3 \end{bmatrix} \quad \text{B} = \begin{bmatrix} a & 0 & 0 \\ b & f & 0 \\ c & d & e \end{bmatrix}$$

- ***Upper Triangular Matrix*** is a square matrix and $a_{ij} = 0$ for $i > j$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Example 7:

$$\text{A} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 3 \end{bmatrix} \text{B} = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$$

EQUALITY OF MATRICES

Two matrices are equal if and only if they have same order and the corresponding elements are also equal.

Example 8:
Determine the values of x and y if:

$$\begin{bmatrix} y & 1 & 3 \\ 5 & 7 & x+2y \end{bmatrix} = \begin{bmatrix} 4-y & 1 & 3 \\ 5 & 7 & y \end{bmatrix}$$

Operations on Matrices

- **Addition And Subtraction Of Matrices**

  For m x n matrices, $A = [a_{ij}]$ and $B = [b_{ij}]$,

  $A + B = C = [c_{ij}]_{mxn}$, where $c_{ij} = a_{ij} + b_{ij}$.

  $A - B = D = [d_{ij}]_{mxn}$, where $d_{ij} = a_{ij} - b_{ij}$.

Note

The addition or subtraction of two matrices with different orders is not defined. We say the two matrices are incompatible.

Example9:

Simplify the given quantity for $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$,

$B = \begin{bmatrix} 4 & 3 \\ -5 & 6 \end{bmatrix}$.

(a) A + B (b) A – B

Scalar Multiplication

If c is a scalar and $A = [a_{ij}]$ then $cA = [b_{ij}]$ where $b_{ij} = ca_{ij}$

Example10:

Let $A = \begin{bmatrix} 1 & 4 \\ 5 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 6 \\ 4 & 2 \end{bmatrix}$. Calculate $3A - 2B$

Properties

(a) $A + B = B + A$ (Commutative)

(b) $(A + B) + C = A + (B + C)$ (Associative)

(c) $A + (-A) = (-A) + A = 0$ (O- zero matrix)

(d) $(\alpha + \beta)A = \alpha A + \beta A$ α, β constant

(e) $\alpha (A + B) = \alpha A + \alpha B$

(f) $\alpha (\beta A) = (\alpha\beta) A$

Multiplication of Matrices

The product of two matrices A and B is defined only when the number of columns in A is equal to the number of rows in B.

If the order of A is $m \times n$ and the order of B is $n \times p$ then AB has order $m \times p$

$$A_{m\times n} B_{n\times p} = AB_{m\times p}$$

$$R = \begin{bmatrix} a_1 & a_2 & a_3 & \dots & a_n \end{bmatrix} \text{and}\, C = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{bmatrix}$$

$$RC = \begin{bmatrix} a_1 b_1 + a_2 b_2 + a_3 b_3 + \dots + a_n b_n \end{bmatrix}$$

Example 11:

Find $\begin{bmatrix} 1 & 2 & 3 \\ -2 & 0 & 5 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -3 & 4 \\ 2 & 1 \end{bmatrix}$

Example 12:

Let $A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$. Show that $AB \neq BA$.

Checking ANSWER using calculator CASIO fx-570MS

STEP1: (to reset all the data)
SHIFT MODE 3 = =

STEP2: (to create matrix A)
MODE MAT 2
SHIFT 4 DIM A
(to key-in the dimension : e.g 2x2)
2 = 2 =
(to key-in the entries, row by row)
-1 = 2 =
3 = 4 =

STEP3: (to create matrix B)
SHIFT 4 DIM B
(to key-in the dimension : e.g 2x2)
2 = 2 =
(to key-in the entries, row by row)
2 = -1 =
3 = 2 =

STEP4: (to multiply 2 matrices)
SHIFT 4 MAT A X SHIFT 4 MAT B =
SHIFT 4 MAT B X SHIFT 4 MAT A =

Properties

(a) $A(BC)=(AB)C$ (Associativite)
(b) $A(B+C)=AB+AC$ (Distributive)

Transpose Matrix

The transpose of a matrix A, written as A^T, is the matrix obtained by interchanging the rows and columns of A. That is, the ith column of A^T is the ith row of A for all i's.
If $A_{m\times n} = \lfloor a_{ij} \rfloor$ then $A^T = \lfloor a_{ji} \rfloor$

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \text{then } A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

Propertiesof the transpose matrix

- $(A \pm B)^T = A^T \pm B^T$
- $(A^T)^T = A$
- $(AB)^T = B^T A^T$
- $(kA)^T = kA^T$

Example 13:

Let $B = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}_{3\times1}$ then $B^T = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}_{1\times3}$

If $D = \begin{bmatrix} 1 & 3 & 3 \\ 2 & 5 & 4 \\ 1 & 3 & 5 \end{bmatrix}_{3\times3}$ then $D^T = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 5 & 3 \\ 3 & 4 & 5 \end{bmatrix}_{3\times3}$

Example 14:

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$.

Show that (a) $(A + B)^T = A^T + B^T$
(b) $(BC)^T = C^T B^T$

Solution

LECTURE 2

TOPIC : 4.0 MATRICES AND SYSTEMS OF LINEAR EQUATIONS

SUBTOPIC: 4.2 Determinant of Matrices

LEARNING OUTCOMES :

At the end of the lesson, students should be able to

(a) Define the minor and cofactor for a_{ij}

(b) Discuss the expansion of the cofactor and find the determinant of a 3 ×3 matrix.

CONTENT

A) Determinant of 2 x 2 Matrices

Given $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Then determinant $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

Example 1:

Given $A=\begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$

B) Determinant of 3 x 3 Matrices

I) Minor and Cofactor

Let A be n x n matrix,

The **minor** M_{ij} of the element a_{ij} is the determinant of the matrix obtained by deleting the *i*th row and *j*th column of A

The **cofactor** C_{ij} of the element a_{ij} is $C_{ij} = (-1)^{i+j} M_{ij}$

The signs of $(-1)^{i+j}$ is corresponding to $\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$

Example 2:

Given $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 4 & 2 \\ 1 & 4 & 3 \end{bmatrix}$

i) Minor

M_{11} is obtained by deleting the first row and first column from A.

$M_{11} = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 4 & 3 \end{vmatrix} = \begin{vmatrix} 4 & 2 \\ 4 & 3 \end{vmatrix} = 4$

Similarly

$$M_{32} = \begin{vmatrix} 1 & 2 & -1 \\ 3 & 4 & 2 \\ 1 & 4 & 3 \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} = 5$$

ii) Cofactor

$$C_{ij} = (-1)^{i+j} M_{ij}$$

Then, $C_{11} = (-1)^{1+1} M_{11} = 4$ and
$C_{32} = (-1)^{3+2} M_{32} = -5$

Example 3:

Given $A = \begin{bmatrix} 2 & -4 & -2 \\ -2 & 0 & 4 \\ 4 & 3 & -3 \end{bmatrix}$

Find:

i. M_{12} and C_{12}

ii. M_{23} and C_{23}

<u>Determinant of 3 x 3 matrix</u>

The value of determinant can be found by using the expansion of cofactor along any row or any column

$$|A| = \Sigma a_{ij} c_{ij}, \quad i = 1,2,\ldots,n \ \ and \ \ j = 1,2,\ldots,n$$

For example, by expanding along the first row

Elements in 1^{st} row : a_{11}, a_{12}, a_{13}

$$|A| = a_{11}c_{11} + a_{12}c_{12} + a_{13}c_{13}$$

By expanding along first column
Elements in 1^{st} column : a_{11}, a_{21}, a_{31}
$|A| = a_{11}c_{11} + a_{21}c_{21} + a_{31}c_{31}$

Example 4:

Let $A = \begin{bmatrix} 3 & -1 & 4 \\ 1 & -2 & 7 \\ 5 & -1 & 10 \end{bmatrix}$, Find $|A|$ by expanding along;

(a) second row
(b) first column

HINT- Choose row or column that has the most zero.

Checking DETERMINANT using calculator CASIO fx-570MS

STEP1: (to reset all the data)
SHIFT MODE 3 = =

STEP2: (to create matrix A)
MODE MAT 2
SHIFT 4 DIM A
(to key-in the dimension : e.g 3X3)
3 = 3 =
(to key-in the entries, row by row)
3 = -1 = 4 =
1 = -2 = 7 =
5 = -1 = 10 =

STEP 3: (to find the determinant)
SHIFT 4 DET
SHIFT 4 MAT A =

Properties of Determinant

1. If all the elements in a row or a column of a square matrix A are zeroes, then |A| = 0.

Example:

a) $A = \begin{bmatrix} 3 & 5 & 2 \\ 0 & 0 & 0 \\ -4 & 2 & -6 \end{bmatrix} \quad |A| = 0$

b) $A = \begin{bmatrix} 4 & -1 & 0 \\ 2 & -3 & 0 \\ 1 & 5 & 0 \end{bmatrix} \quad |A| = 0$

2. If any 2 rows or 2 columns of a square matrix A are identical, then |A| = 0

Example:

a) $A = \begin{bmatrix} 2 & 5 & 8 \\ -3 & 1 & 6 \\ 2 & 5 & 8 \end{bmatrix} \quad |A| = 0$

b) $A = \begin{bmatrix} 4 & 4 & 2 \\ 3 & 3 & -6 \\ 1 & 1 & 7 \end{bmatrix} \quad |A| = 0$

3. If A is a square matrix, then $|A| = |A^T|$.

   Example:

   By using $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -1 \\ -2 & -1 & 1 \end{bmatrix}$, verify that $|A| = |A^T|$.

   Solution:

$$|A| = (1)\begin{vmatrix} 3 & -1 \\ -1 & 1 \end{vmatrix} - (-1)\begin{vmatrix} 0 & -1 \\ -2 & 1 \end{vmatrix} + (2)\begin{vmatrix} 0 & 3 \\ -2 & -1 \end{vmatrix}$$

$$= (3-1) + 1(0-2) + 2(0-(-6))$$

$$= 2 - 2 + 2(6)$$

$$= 12$$

$$A^T = \begin{bmatrix} 1 & 0 & -2 \\ -1 & 3 & -1 \\ 2 & -1 & 1 \end{bmatrix}$$

$$|A^T| = (1)\begin{vmatrix} 3 & -1 \\ -1 & 1 \end{vmatrix} - 0 + (-2)\begin{vmatrix} -1 & 3 \\ 2 & -1 \end{vmatrix}$$

$$= (3-1) + 0 - 2(1-6)$$

$$= 12$$

$|A| = |A^T|$.

4. If A is an upper triangular or a lower triangular matrix, then $|A|$ can be obtained by multiplying the elements on the leading diagonal.

   Example:

a) $A = \begin{bmatrix} 1 & 4 & -3 \\ 0 & -2 & 5 \\ 0 & 0 & -1 \end{bmatrix}$ is an upper triangular matrix

$|A| = (1)(-2)(-1)$
$= 2$

b) $A = \begin{bmatrix} 3 & 0 & 0 \\ 4 & -5 & 0 \\ 6 & 8 & -2 \end{bmatrix}$ is a lower triangular matrix

$|A| = (3)(-5)(-2)$
$= 30$

5. If A and B are two matrices of the same order, then |AB|=|A| |B|

   Example:

$$A = \begin{bmatrix} 1 & -1 & 4 \\ 0 & 2 & 3 \\ 6 & -2 & 12 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 5 & 2 \\ 0 & 1 & 1 \end{bmatrix} \quad AB = \begin{bmatrix} 3 & 0 & 2 \\ -2 & 13 & 7 \\ 14 & 8 & 8 \end{bmatrix}$$

$$|A| = -36 \quad |B| = 7 \quad |AB| = (-36)(7) = -252$$

6. If a square matrix B is obtained from a square matrix A by interchanging any 2 rows or any 2 columns, then |B|= -|A|

   Example:

Let $A=\begin{bmatrix}3 & 0 & 1\\ 2 & 1 & 4\\ 3 & 2 & 5\end{bmatrix}$ $B=\begin{bmatrix}2 & 1 & 4\\ 3 & 0 & 1\\ 3 & 2 & 5\end{bmatrix}$

$$|A|=-8$$
$$|B|=-|A|=8$$

7. If a square matrix B is obtained from a square matrix A by multiplying each element of any row or any column of matrix A by a constant k, then |B|=k|A|

Example:

Given that $A=\begin{bmatrix}2 & 4 & 0\\ 1 & -3 & 1\\ 5 & 2 & 0\end{bmatrix}$ $B=\begin{bmatrix}2 & 4 & 0\\ 3 & -9 & 3\\ 5 & 2 & 0\end{bmatrix}$

$$C=\begin{bmatrix}10 & 4 & 0\\ 5 & -3 & 1\\ 25 & 2 & 0\end{bmatrix}$$

a) Show that $|A|=16$

$$\begin{aligned}|A| &= (2)\begin{vmatrix}-3 & 1\\ 2 & 0\end{vmatrix}-(4)\begin{vmatrix}1 & 1\\ 5 & 0\end{vmatrix}+0\\ &= 2(0-2)-4(0-5)+0\\ &= -4+20=16\end{aligned}$$

b) By using the result in (a), find the values of $|B|$ and $|C|$

$$B = \begin{bmatrix} 2 & 4 & 0 \\ 3 & -9 & 3 \\ 5 & 2 & 0 \end{bmatrix} = 3\begin{bmatrix} 2 & 4 & 0 \\ 1 & -3 & 1 \\ 5 & 2 & 0 \end{bmatrix}$$

$$= 3A$$

$$|B| = 3|A|$$

$$= 3(16) = 48$$

$$C = \begin{bmatrix} 10 & 4 & 0 \\ 5 & -3 & 1 \\ 25 & 2 & 0 \end{bmatrix} = 5\begin{bmatrix} 2 & 4 & 0 \\ 1 & -3 & 1 \\ 5 & 2 & 0 \end{bmatrix}$$

$$= 5A$$

$$|C| = 5|A|$$

$$= 5(16) = 80$$

LECTURE 3

TOPIC : 4.0 MATRICES AND SYSTEMS OF LINEAR EQUATIONS

SUBTOPIC: 4.3 Inverse Matrices

LEARNING OUTCOMES:

At the end of the lesson, students should be able to:

(a) Find the adjoint of matrix A

(b) Define the inverse of a matrix

(c) Find the inverse matrix using the adjoint matrix

CONTENT

Adjoint Matrix

Let $C = [c_{ij}]$ be the cofactor matrix of A.
Adjoint of matrix A (*adj A)* is defined as the transpose of the cofactor matrix that is

$$adj\ A = C^T = [c_{ij}]^T = [c_{ji}]$$

Remember: Cofactor $, c_{ij}$ = (-1)$i+jm_{ij}$,

$$C = \begin{bmatrix} +m_{11} & -m_{12} & +m_{13} \\ -m_{21} & +m_{22} & -m_{23} \\ +m_{31} & -m_{32} & +m_{33} \end{bmatrix}$$

Example 1:

Given $A=\begin{bmatrix}1 & 2 & 3\\ 3 & 2 & 1\\ 1 & 1 & 3\end{bmatrix}$. Find the adjoint of A.

Example 2:

Given $P=\begin{bmatrix}1 & 2 & 2\\ 2 & 10 & 5\\ 1 & 3 & 3\end{bmatrix}$. Find the adjoint of P.

Inverse Matrices

The inverse of a matrix A is denoted by A^{-1}, given that $|A| \neq \frac{1}{A}$. **(Remember!!!!! $A^{-1} \neq \frac{1}{A}$)**

Therefore $AA^{-1} = A^{-1}A = I$.

There are **<u>3 methods</u>** to obtain inverse of matrices:

(a) Adjoint Method $A^{-1} = \frac{1}{|A|} adj A$

(b) Use the property $AB = k\mathrm{I}$

(c) Elementary Row Operations (ERO)

(A) Finding Inverse By Using Adjoint Method

The inverse of a matrix A is denoted by A^{-1}, given that $|A| \neq 0$.

If $|A| \neq 0$,,

~ A is a non-singular matrix
~ Inverse matrix exists

If $|A| = 0$,,

~ A is a singular matrix
~ Inverse matrix does not exist

(i) Inverse of a 2 x 2 matrix

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then A^{-1} is given by

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Example 3:

Find the inverse matrix for $A = \begin{bmatrix} 3 & 1 \\ 5 & 4 \end{bmatrix}$

(ii) Inverse of a 3 x 3 matrix

Example 4:

Find the inverse matrix of $B = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 2 & 2 \\ -2 & -1 & 0 \end{bmatrix}$

Checking INVERSE MATRIX using calculator CASIO fx-570MS

STEP1: (to reset all the data)
SHIFT MODE 3 = =

STEP2: (to create matrix A)
MODE MAT 2
SHIFT 4 DIM A
(to key-in the dimension : e.g 3X3)
3 = 3 =
(to key-in the entries, row by row)
1 = 3 = 2 =
0 = 2 = 2 =
-2 = -1 = 0 =

STEP3: (to find the inverse)
SHIFT 4 MAT A X^{-1}

Properties of Inverse Matrix

$*(A^{-1})^{-1} = A$

$*(A^{T})^{-1} = (A^{-1})^{T}$

$*(AB)^{-1} = B^{-1}A^{-1}$

$*\left|A^{-1}\right| = \dfrac{1}{|A|}$

(B) Using the property $AB = kI$

If $AB = BA = I$

$\Rightarrow B = A^{-1} \ \ and \ \ A = B^{-1}$

$Hence\ ,\ if \ \ AB = \alpha I$

$$then \ \ A^{-1} = \frac{1}{\alpha}B$$

$$and \ \ also \ \ B^{-1} = \frac{1}{\alpha}A$$

If $AB = I$ where A and B are square matrices, then B is called the inverse matrix of A and is written as A^{-1}. Thus $AA^{-1} = A^{-1}A = I$

Example 5:

Given $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 5 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & -1 \\ -10 & 4 & 2 \\ 7 & -3 & -1 \end{bmatrix}$. It is known that $AB=kI$, where k is a constant and I is an $3X3$ matrix. Find k and hence deduce A^{-1}.

Example 6:

Given $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$ such that $A^2 - 7A + 10I = 0$

where I is $3x3$ identity matrix and 0 is zero matrix. Obtain A^{-1} and show that $A^3 = 39A - 70$.

LECTURE 4

TOPIC : 4.0 MATRICES AND SYSTEMS OF LINEAR EQUATIONS

SUBTOPIC: 4.3 Inverse Matrices

LEARNING OUTCOMES:

At the end of the lesson, students should be able to:
(a) apply the elementary row operations to obtain the inverse of 2 x 2 matrix.
(b) apply the elementary row operations to obtain the inverse of 3 x 3 matrix.

SET INDUCTION

There are 3 methods to obtain inverse of matrices:

(a) Adjoint Method $A^{-1} = \dfrac{1}{|A|} adj\ A$

(Remember! $A^{-1} \neq \dfrac{1}{A}$)

(b) Use the property $AB = kI$

(c) Elementary Row Operations (ERO)

CONTENT

Finding Inverse Using Elementary Row Operations.

Given an augmented matrix [A | I] with rows Ri, i=1,2,…,m.

The elementary row operations include the following operations:

(a) Interchanging the i^{th} row and j^{th}row. $R_i \leftrightarrow R_j$

(b) Multiplying the i^{th}row with a nonzero constant.
$R_i^* = \alpha R_j$ (* : is the new row)

(c) Adding a multiple of j^{th} row to the i^{th} row.
$R_i^* = \alpha R_j + R_i$

PROCEDURE

STEP 1: Obtain a 1 in the first position on the leading diagonal.

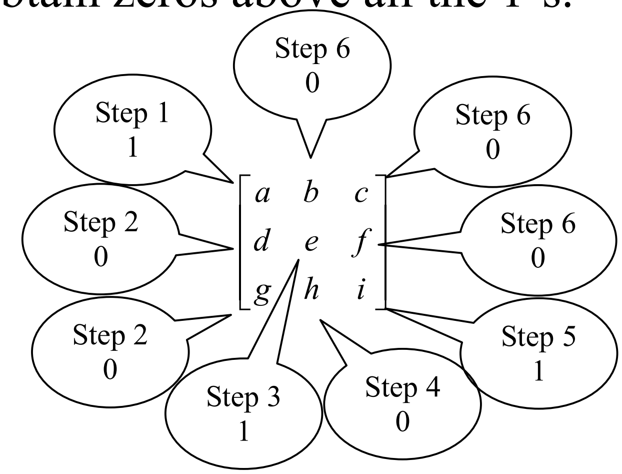
STEP 2: Obtain zeros under 1 in the first column.

STEP 3: Obtain a 1 the second position on the leading diagonal.

STEP 4: Obtain a zero under 1 in the second column.

STEP 5: Obtain a 1 in the third position on the leading diagonal.

STEP 6: Obtain zeros above all the 1's.



NOTES:[A | I]$\Rightarrow$ [I | A^{-1}] – As matrix A changes to the Identity matrix, the augmented Identity matrix changes to the Inverse of matrix A.

Example 1:

Find the inverse of the given matrix using elementary row operations.

(a) $\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

Inverse Matrices

Example 2:

Given $P = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$. Find the inverse of P by using ERO.

Example 3:

If $B = \begin{bmatrix} 2 & -4 & 3 \\ 1 & 2 & -1 \\ -2 & -3 & 1 \end{bmatrix}$. Find B^{-1}.

LECTURE 5

TOPIC : 4.0 MATRICES AND SYSTEMS OF LINEAR EQUATIONS

SUBTOPIC: 4.4 System Of Linear Equations With Three Variables

LEARNING OUTCOMES:

At the end of the lesson, students should be able to:
(a) Solve $AX=B$ using Inverse Matrix.

SET INDUCTION

Definition – Assuming we have a square matrix A, which is non-singular (det(A) does not equal to zero , then there exists an nxn matrix A^{-1} which is called the inverse of A, such that this property holds:

$AA^{-1} = A^{-1}A = I$ where I is the identity matrix.

Using the Inverse Matrix to solve $AX = B$

If A is a n x n square matrix that has an inverse A^{-1}, X is a variable matrix and B ia a known matrix, both with n rows, then the solution of matrix equations $AX = B$ is given by

$$X = A^{-1} B$$

Proof :$A X = B$ (3 x 3 square matrix)

$$A^{-1}(A X) = A^{-1} B$$

$$(A^{-1}A)X = A^{-1}B$$

$$IX = A^{-1}B$$
$$X = A^{-1}B$$

Example 1:

Solve the following system of equations by using the inverse matrix

$$3x_1 + x_2 + 2x_3 = 11$$
$$3x_1 + 2x_2 + 2x_3 = 10$$
$$x_1 + x_3 = 5$$

Example 2:

Given $A = \begin{bmatrix} 1 & 3 & -9 \\ 5 & -1 & 3 \\ 1 & 3 & 7 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 3 & 0 \\ 2 & -1 & 3 \\ -1 & 0 & 1 \end{bmatrix}$

Find AB and A^{-1}. Hence, solve the following linear equations.

$$x + 3y - 9z = 7$$
$$5x - y + 3z = 5$$
$$x + 3y + 7z = 1$$

LECTURE 6

TOPIC : 4.0 MATRICES AND SYSTEMS OF LINEAR EQUATIONS

SUBTOPIC:4.4 System Of Linear Equations With Three Variables

LEARNING OUTCOMES:

At the end of the lesson, students should be able to:
(a) Solve $AX=B$ using Gauss-Jordan Elimination method

SET INDUCTION

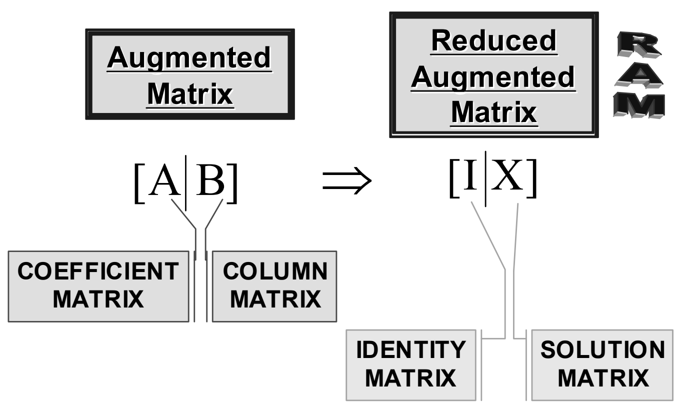
In the Gauss-Jordan Elimination Method our goal is to use row operations to transform an augmented matrix into a reduced form. It can be shown that any linear system must have exactly one solution, no solution, or an infinite number of solutions, regardless of the number of equations or the number of variables in the system. The terms unique, consistent, and inconsistent are used to describe these solutions, just as they are for systems with three variables.
The Gauss-Jordan elimination method is to used to determine these types of solutions according to the reduced augmented matrix.

CONTENT

The Gauss- Jordan Elimination Method

1. Form the augmented matrix whose first n columns constitute A and whose last columns form B , symbolically $\left[A \mid B\right]$.

2. The elementary row operation are used to reduce the augmented matrix to form a Reduced Augmented Matrix (RAM).



Example 1:

$$x + 2y + 3z = -1$$
$$x - 3y - 5z = 6$$
$$x + 2y - z = -5$$

Example 2:
Solve the following system of equations using the Gauss- Jordan Eliminationmethod

$$x - 3y + z = -12$$
$$x + y + z = 0$$
$$2x - y + z = -8$$

Example 3:
Ali, Bob and Ravi bought tickets for three separate performances. The table below shows the number of tickets bought by each of them.

	concert	orchestral	opera
Ali	2	1	1
Bob	1	1	1
Ravi	2	2	1

(a) If the total cost for Ali was RM 122, for Bob RM 87 and for Ravi RM 146, represent this information in the form of three equations.
(b) Find the cost per ticket for each of the performances using G-J elimination method.

(c) Determine how much it would cost Hassan to purchase 2 concert, 1 orchestral and 3 opera tickets.